

Hexagon Desks

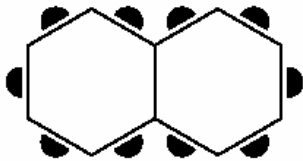
This problem gives you the chance to:

- find and extend a number pattern
- plot and use a graph

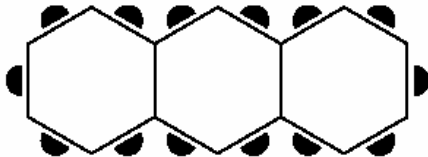
Sarah finds how many students can sit around a row of desks. The top surface of each desk is a hexagon, and the hexagons are arranged in rows of different shapes.



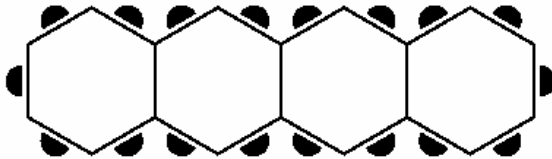
1 desk 6 students



2 desks 10 students



3 desks 14 students

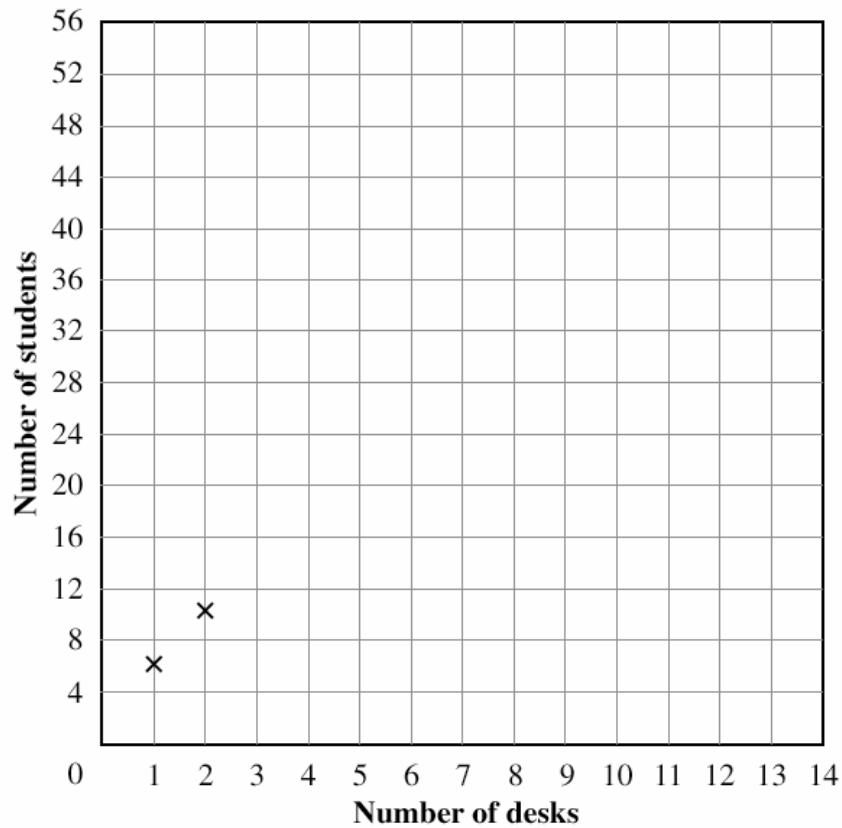


4 desks

1. Complete Sarah's table.

Number of desks in a row	Number of students
1	6
2	10
3	
4	
5	
6	

2. On the grid, plot the results from the table you completed in question 1. The first two points have already been plotted for you.



3. Sarah says that 47 students can sit around a row of 11 desks. Without drawing the desks, explain how you know that Sarah is wrong.

How many students can sit around a row of 11 desks? _____

10

Hexagon Desks

Test 4 Form A Rubric

The core elements of performance required by this task are:

- find and extend a number pattern
- plot and use a graph

Based on these, credit for specific aspects of performance should be assigned as follows:

	Points	Section Points														
<p>1. Correctly completes the table:</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Number of desks in a row</th> <th>Number of students</th> </tr> </thead> <tbody> <tr><td>1</td><td>6</td></tr> <tr><td>2</td><td>10</td></tr> <tr><td>3</td><td>14</td></tr> <tr><td>4</td><td>18</td></tr> <tr><td>5</td><td>22</td></tr> <tr><td>6</td><td>26</td></tr> </tbody> </table> <p>All four correct values: 2 points</p> <p><i>Partial credit:</i> Three or two correct values: 1 point</p>	Number of desks in a row	Number of students	1	6	2	10	3	14	4	18	5	22	6	26	<p>2</p> <p>(1)</p>	2
Number of desks in a row	Number of students															
1	6															
2	10															
3	14															
4	18															
5	22															
6	26															
<p>2. Correctly plots the four values from the student's answer to question 1 on the grid:</p> <p><i>Accept points in the correct square that are not on the horizontal grid lines.</i></p>	<p>4 × 1 ft</p>	4														
<p>3. Correctly continues table or graph.</p> <p>or</p> <p>States that 47 is not an even number.</p> <p>or</p> <p>Gives a correct alternative reason.</p> <p>Gives correct answer as:</p> <p>46 students</p>	<p>2</p> <p>or</p> <p>2</p> <p>or</p> <p>2</p> <p>2</p>	4														
Total Points		10														

Looking at Student Work - Hexagon Desks

In general students did very well on Hexagon Desks. Many 4th grade students showed a good understanding of algebraic thinking and functions. Student A expresses a rule in part 3 that could translate directly into a formula, $2(x-2) + 10$.

Student A

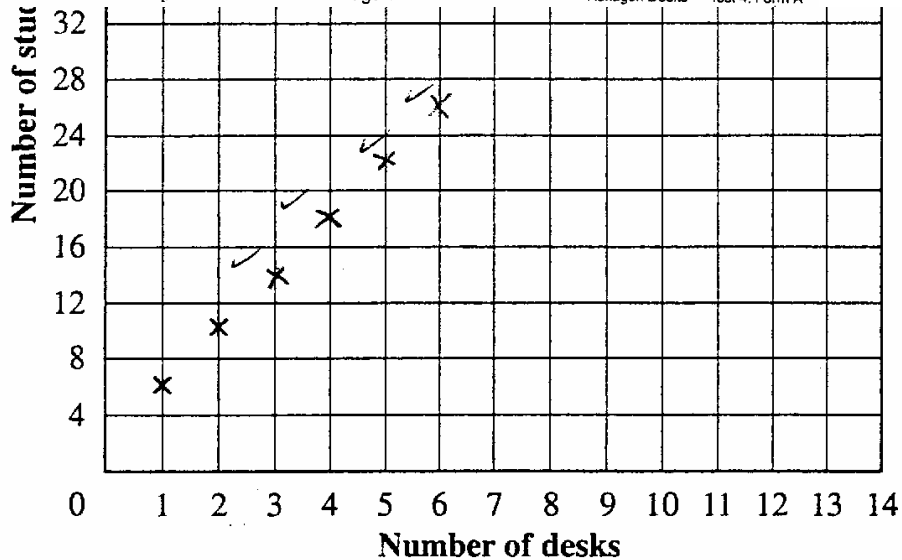
Number of desks in a row	Number of students
1	6
2	10
3	14 ✓
4	18 ✓
5	22 ✓
6	26 ✓

2

Published by CTB/McGraw-Hill LLC. Copyright © 2003
by Mathematics Assessment Resource Service. All rights reserved.

Page 4

Hexagon Desks Test 4: Form A



Sarah says that 47 students can sit around a row of 11 desks.
Without drawing the desks, explain how you know that Sarah is wrong.

At each middle tabel there are 4 ✓
at each end tabel there are 5
there are 2 end tabels and 9
middle tabels so that = 46.

How many students can sit around a row of 11 desks?

46 ✓

Student B does a great job of explaining why adding a new table gives four more seats each time. The student is paying attention to how the pattern grows, what elements are growing and how they fit together, which is the logic underlying a generalization or formula.

Student B

Without drawing the desks, explain how you know that Sarah is wrong.

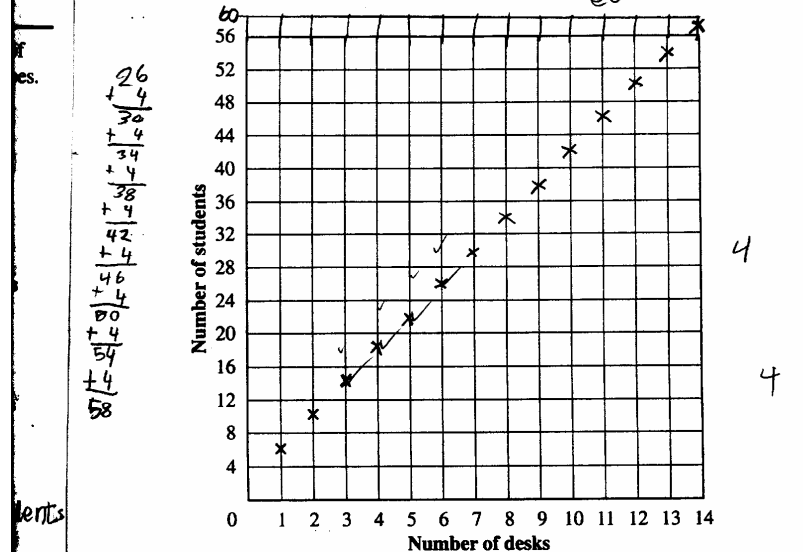
I know that Sarah is wrong because the number of students sitting in any number of desks all have to be even because when you add a table you're adding five and taking one off so you're actually adding four.

How many students can sit around a row of 11 desks?

46 students ✓

Student C has a complete and simplified formula for this situation, but errs in trying to make her own notation. She knows that adding four every time is the same as multiplying by four and that in the first case there is an extra 2. Her rule is multiply by 4 and add 2 that equates to $4x+2$. She cannot however put that rule clearly in the form of a number sentence or precise words. The graph has been made by adding four to get each point. The work for 46 is shown next to the writing for part 3.

The first two points have already been plotted for you. etc.



3. Sarah says that 47 students can sit around a row of 11 desks. Without drawing the desks, explain how you know that Sarah is wrong.

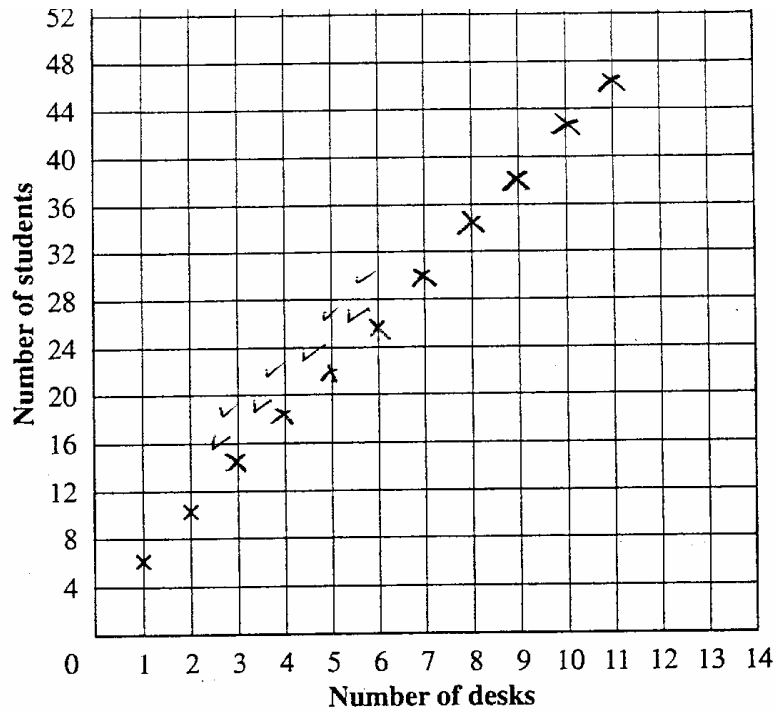
she is wrong because $11 \times 4 = 44$ not 47. what made me come up with $11 \times 4 = 44$ is each time they are adding by 4 and then add 2

How many students can sit around a row of 11 desks?

46 ✓

Student D is part way to this same understanding. The student knows that the first one has an extra 2, but forgets to subtract out the 4 for desk 12. (The procedure should have been amount for 6 desks + the amount for 6 desks - 4 to get amount for 11 desks - 2 for the extra in the first desk.)

Student D



4

4

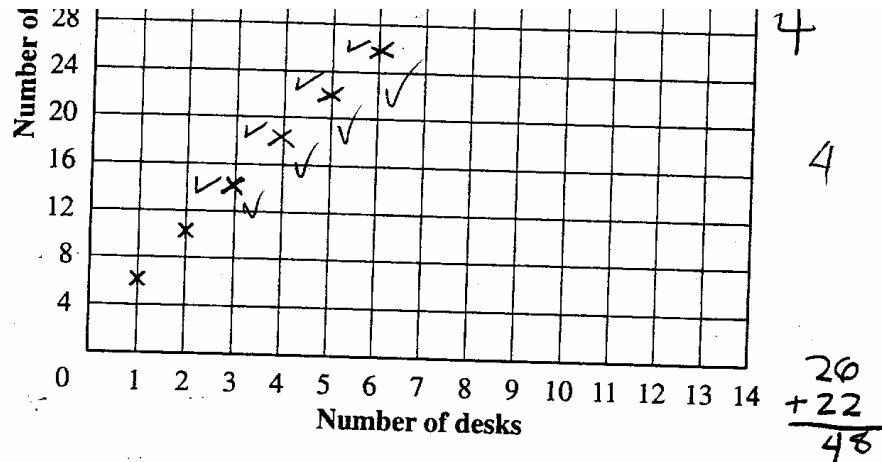
3. Sarah says that 47 students can sit around a row of 11 desks.

Without drawing the desks, explain how you know that Sarah is wrong.

Since I know 6 desks = 26 desks I multiplied $(2 \times 26) - 2 = 50$

Student E forgets about the 2 extra for the first desk.

Student E



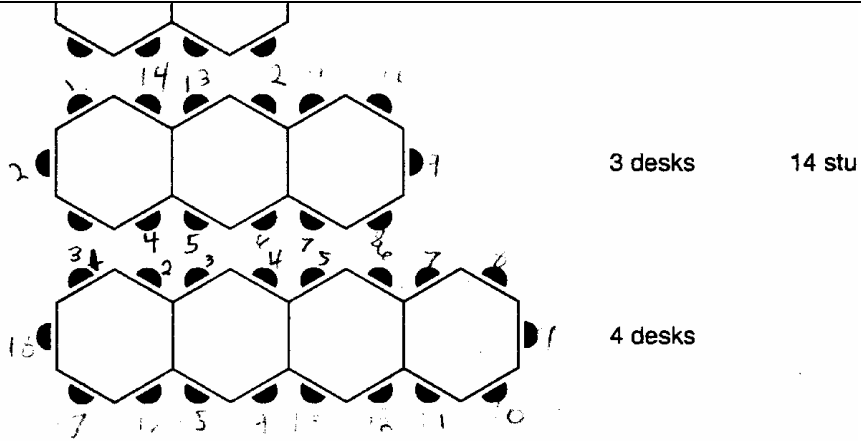
3. Sarah says that 47 students can sit around a row of 11 desks. Without drawing the desks, explain how you know that Sarah is wrong.

Sarah is wrong because on the page before 5 desks is 22, and 6 desks is 26, $6+5=11$ desks $22+26=48$, 48 is 1 more than what she thought so she's wrong.

How many students can sit around a row of 11 desks?

48 students

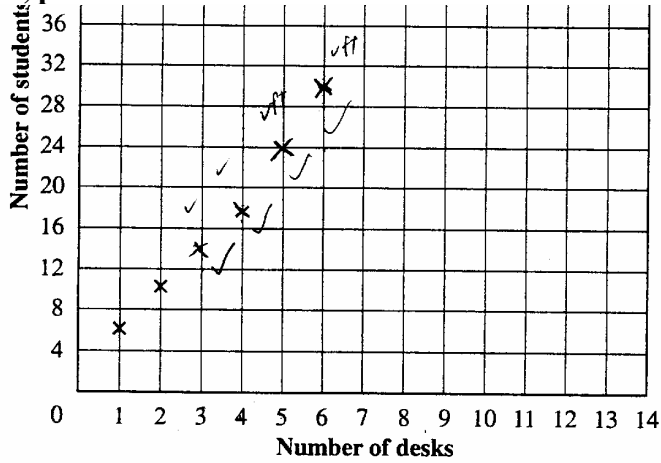
Student F makes some errors on the table, adding 4's and then adding 6's. But the student's procedure in 3 would have been correct if the answer for desk 6 in the table had been correct. If you double the amount for the sixth term and subtract out the first term (6) which includes the constant, you will have the amount for 11 (6+5).



1. Complete Sarah's table.

Number of desks in a row	Number of students
1	6
2	10
3	14 ✓
4	18 ✓
5	24 +
6	30 +

Student F, part 2



3. Sarah says that 47 students can sit around a row of 11 desks.
Without drawing the desks, explain how you know that Sarah is wrong.

sarah is wrong because on the
chart 6 is 30 and 11 would be
more and it is odd not even.

How many students can sit around a row of 11 desks?

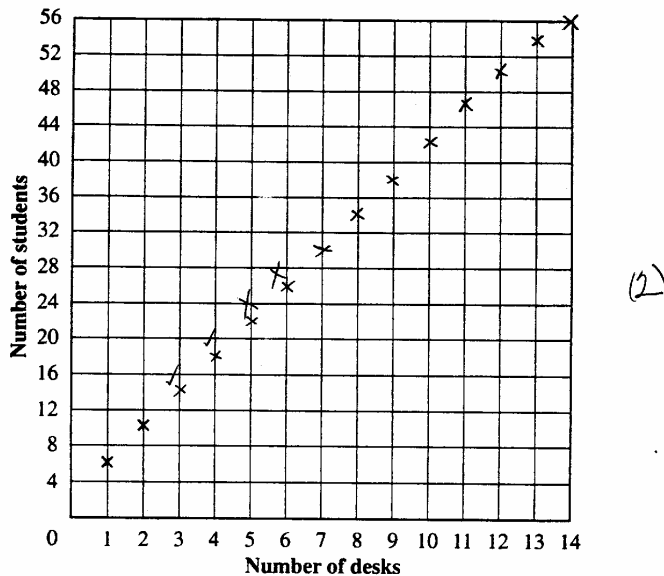
54

$$\begin{array}{r} 30 \\ \times 30 \\ \hline 00 \\ 900 \\ \hline 5400 \end{array}$$

Many students seem to have a firm belief that the points in a graph will lie in a straight line. Student G has the wrong numbers in the table, but shows a graph that matches the correct values for this function. The student does not use the graph to help answer the question, but relies on a drawing and counting strategy. This strategy is not successful because the numbers are large and cumbersome.

complete Sarah's table.

Number of desks in a row	Number of students
1	6
2	10
3	14 ✓
4	18 ✓
5	22 ✗
6	28 ✗



3. Sarah says that 47 students can sit around a row of 11 desks. Without drawing the desks, explain how you know that Sarah is wrong.

Sarah is wrong because there could not be 47 students in 11 rows shes add 3 more and thats not the right answer the answer is 44 students. ✗

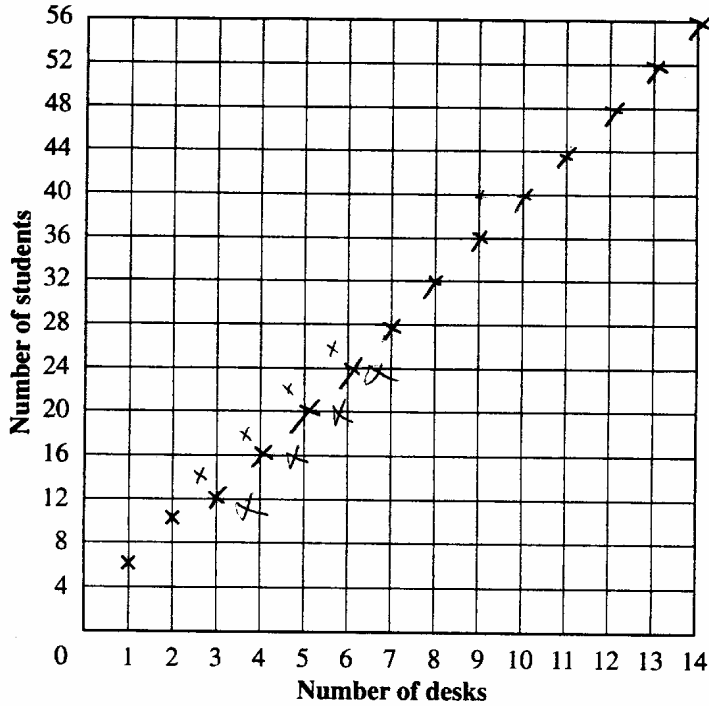
How many students can sit around a row of 11 desks?



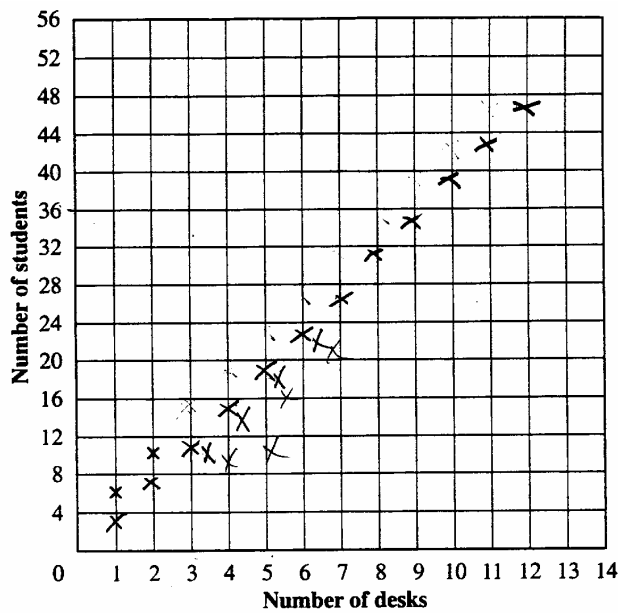
44 students
✗
10

Other students think that the points on the graph must be in a straight line and the straight line must start at 0. See the graphs of Student I and Student J.

Student I



Student J



3. Sarah says that 47 students can sit around a row of 11 desks. Without drawing the desks, explain how you know that Sarah is wrong.

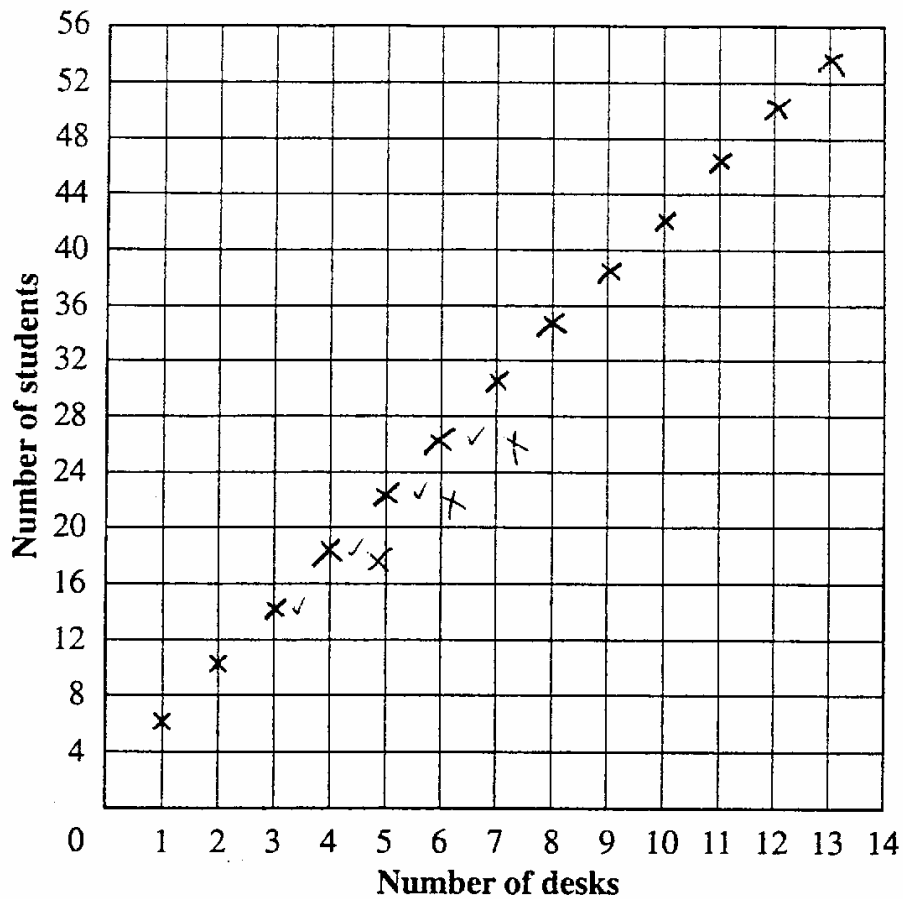
She did not start from 0. $x = 11$ $y = 47$
 0 $x = 0$ $y = 0$

Students did not use their graphs as sense-making tools. Student K saw an incorrect pattern of adding 5's in the table. Because the student thought points should be in a straight line, his graph actually fits the function of desks in a row. Had he used the graph in part 3, Student K could have gotten the correct answer.

Student K

Number of desks in a row	Number of students
1	6
2	10
3	15 x
4	20 x
5	25 x
6	30 x

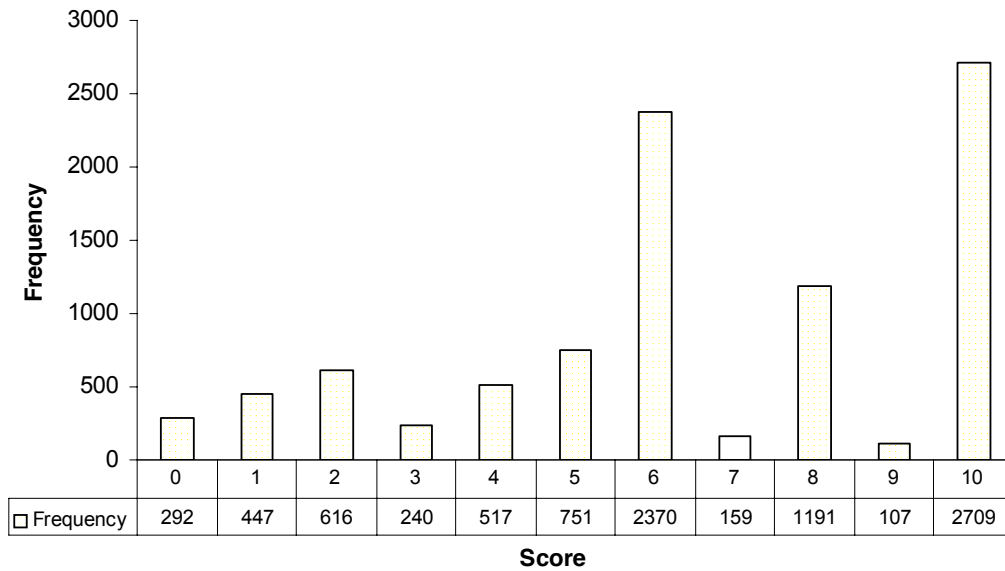
© TBMcGraw-Hill LLC. Copyright © 2003



Grade 4 – Hexagon Desks

Hexagon Desks

Mean: 6.50, S.D.: 2.98



Score:	0	1	2	3	4	5	6	7	8	9	10
% < =	3.1%	7.9%	14.4%	17.0%	22.5%	30.5%	55.7%	57.4%	70.0%	71.2%	100.0%
% > =	100.0%	96.9%	92.1%	85.6%	83.0%	77.5%	69.5%	44.3%	42.6%	30.0%	28.8%

The maximum score available on this task is 10 points.
The cut score for a level 3 response is 5 points.

Most students (about 85%) could find and continue the pattern correctly for at least 2 points and earn partial credit for graphing. Many students (about 70%) could find all the numbers in the table and graph them correctly. 30% of the students could meet all the demands of the task including making a mathematical justification for why 47 students could not fit at 11 desks. About 3% of the students scored no points on this task.

Hexagon Desks

Points	Understandings	Misunderstandings
0	Most students with this score tried the problem.	Students made errors in the pattern, usually adding 5 or 6 each time instead of 4. They could not graph their points correctly. They may have extended the line correctly from the given 2 pts on the grid. They did not make a connection between the values in the table and the points in the graph.
2	Students could correctly continue the table.	Some students thought that the points should line up with zero, so they graphed a pattern that did not match the numbers in their table. Many students at this score point did not attempt the graph.
5	Students could fill in 2 or 3 points in the table and make a correct graph for their points.	
6	Students could continue the table and graph their points correctly.	Most students tried to justify why 47 was not possible because it was not a multiple of 4 (10%).
8	Students could find the correct number of students to sit at 11 desks or explain why 47 was incorrect. Of the students who could explain why 47 was incorrect- most used an argument about even numbers, referred to their graph, or added 4's.	Students had difficulty with justification, they made arguments like "because 46 students fit" instead of process or logic of how they found the 46.
10	Students could continue a pattern in a table, graph the points in a table, use the pattern to justify why 47 students could not sit at an arrangement of 11 desks, and find the correct number of students who could sit at 11 desks.	

Teacher Notes:

Based on teacher observations, this is what fourth grade students seemed to know and be able to do:

- Continue a pattern in a table
- Graph points on a grid
- Understand the relationship between repeated addition and multiplication

Areas of difficulty for fourth graders, fourth grade students struggled with :

- Relating the numbers in the tables to the points on the grid (many students seemed to be relying on a rule about putting points in a straight line and sometimes thinking the straight line needed to go through zero)
- Connecting the table and the graph as having the same information and being related rather than two separate tasks
- Making sense of the graph, only 14% used the graph to help them make a mathematical argument in part 3
- Making a mathematical argument to explain why 47 students can't fit at the desks

Questions for Reflection on Hexagon Desks:

- Could most of your students continue the pattern in the table? How many tried to add on some number other than 4 to make their pattern? What do you think they were seeing or paying attention to?
- How many students had graphs that matched the points in their table? When students made an error in the table, did their graphs still go in a straight line or did they graph the data points in the table?
- What types of experiences have students had this year with graphing? Have they inferred things about graphing that aren't true for all problems, like the lines start at zero? What are some interesting problems you can give students to overcome these misconceptions?
- Students need to see the purpose for making the graph, to use it to help solve problems and to go beyond what they know. What kinds of questions will help promote this kind of thinking by students?
- How many of your students could make rules in part 3 that really matched the situation of the desks, describing what is changing and what is staying the same?
- Did your students have difficulty placing the plotted points? Are they used to working with graphs with different vertical scales?

Teacher Notes:

Instructional Implications:

Students at this grade level need to experience analyzing growing patterns. They also need to be able to correctly plot the information from a table onto a graph with pre-established scales. While many students seemed to know that the plots for growing patterns should be on a straight line, the ordered numbers on their graphs did not match the numbers in their tables. Students need more experiences with linear relationships that are not proportional: i.e., the numbers are not multiples. For example, in this problem the number of students increases by four with each additional desk, but the total number of students is not a multiple of four, so this is not a proportional relationship. Students also need to be able to work backward from a solution. Students might do this by extending the table, extending their graph, or by using repeated addition. An important part of patterning is to recognize the relationship between the two variables. Learning to think and describe what is changing and what is staying the same helps students to find rules that will work for all cases.

Teacher Notes:
