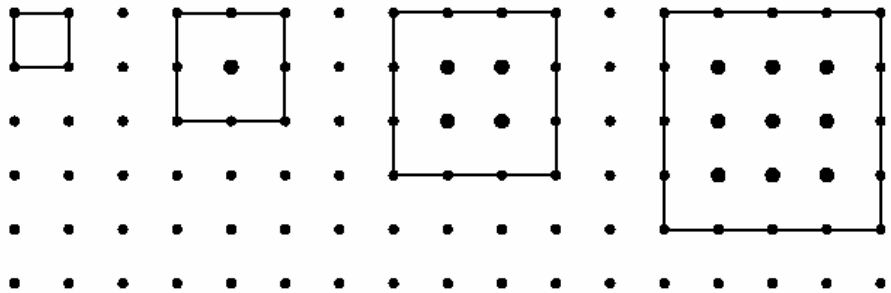
8th grade Task 4 Dots and Squares

Student Task	Find and table number patterns in a geometric content. Find and use rules or formulas to answer questions.
Core Idea 3 Algebra and Functions	**Understand relations and functions, analyze mathematical situations, and use models to solve problems involving quantity and change.** • Use tables to analyze the nature of changes on quantities in linear relationships • Recognize and generate equivalent forms of simple algebraic expressions and solve linear equations. • Represent, analyze, and generalize a linear relationship (7^{th} grade) • Use symbolic algebra to represent situations to solve problems (7^{th} grade)
Core Idea 2 Mathematical Reasoning	**Employ forms of mathematical reasoning and justification appropriately to the solution of a problem.** • Use mathematical language and representations to make situations easier to understand

Dots and Squares

This problem gives you the chance to:
- tabulate and find number patterns in a geometric context
- find and use rules or formulas

Sally draws squares of different sizes and counts the dots inside each square.



Sally makes a table showing the length of one side of each square (S), the perimeter of each square (P), and the number of dots inside each square (I).

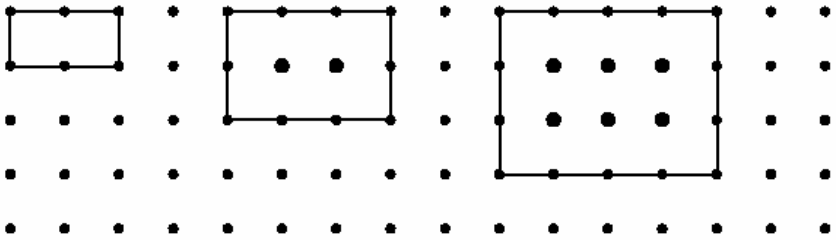
S	1	2	3	4	5	6
P	4	8	12	16		
I	0	1	4	9		

1. Fill in the empty boxes in Sally's table.

2. Write a rule or a formula for finding the number of dots inside a square when you know the length of a side of the square.

3. There are 49 dots inside a square. What is the length of one side of the square? Explain your reasoning.

Tom draws rectangles and counts the dots inside.



He makes a table showing the length of each rectangle (L), the width of each rectangle (W), and the number of dots inside (I).

L (in squares)	2	3	4	5	6
W (in squares)	1	2	3	4	5
I	0	2	6		

4. Fill in the empty boxes in the table above.

5. Write a rule or formula for finding the number of dots inside a rectangle (I) when you know the length (L) and the width (W) of the rectangle.

_____

_____

6. There are 63 dots inside a rectangle.
What is the length of the rectangle? _____

What is the width of the rectangle? _____

Dots and Squares — Test 8 Form A Rubric

The core elements of performance required by this task are:
- tabulate and find number patterns in a geometric context
- find and use rules or formulas

Based on these, credit for specific aspects of performance should be assigned as follows:

	Points	Section Points
1. Correctly completes the table: **S** 1 2 3 4 5 6 **P** 4 8 12 16 **20** **24** **I** 0 1 4 9 **16** **25** *Allow 1 point for each two correct values.*	2 × 1	2
2. Gives correct answer as: $\mathbf{I = (S - 1)^2}$ *Accept verbal equivalents.*	2	2
3. Gives correct answer such as: The length of the side of the square is **8**. Gives explanation such as: $49 = 7^2$	1 1	2
4. Correctly completes the table: **L (in squares)** 2 3 4 5 6 **W (in squares)** 1 2 3 4 5 **I** 0 2 6 **12** **20**	2 × 1	2
5. Gives correct answer as: **I = (W − 1)(L − 1)** (or equivalent) *Accept verbal equivalents.*	1	1
6. Gives correct answer as: The length of the rectangle is 10. *(accept 22 or 64)* The width of the rectangle is 8. *(accept 4 or 2)* *Accept 63 = 9 × 7 or 21 × 3 or 63 × 1.* Both answers correct.	1	1
Total Points		**10**

Looking at Student Work – Dots and Squares

Dots and Squares requires students to identify patterns and generalize the patterns in the form of a rule or formula. Student A looks at the relationship between the geometric pattern and how it affects the relationship between the shapes and the number patterns. Student A is able to find a rule for both part 2 and part 5 of the task. The rule for part 5 will work for any size rectangle and is not restricted to rectangles where the length and width vary by one unit.

Sally makes a table showing the length of one side of each square (S), the perimeter of each square (P), and the number of dots inside each square (I).

S	1	2	3	4	5	6
P	4	8	12	16	20 ✓	24 ✓
I	0	1	4	9	16 ✓	25 ✓

2
2

✓ ✓

1. **Fill in the empty boxes in Sally's table.**

2. **Write a rule or a formula for finding the number of dots inside a square when you know the length of a side of the square.**

(side of square − 1)^2 = # of dots ✓ 2

2

3. **There are 49 dots inside a square. What is the length of one side of the square? Explain your reasoning.**

1

8 is the length of one side because the square root of 49 is 7, plus 1 is 8. ✓ 1 ✓ 2

L (in squares)	2	3	4	5	6
W (in squares)	1	2	3	4	5
I	0	2	6	12 ✓	20 ✓

1 2

✓ ✓

4. **Fill in the empty boxes in the table above.**

5. **Write a rule or formula for finding the number of dots inside a rectangle (I) when you know the length (L) and the width (W) of the rectangle.**

(L−1)(W−1)=I ✓ ✓ 1

1

6. **There are 63 dots inside a rectangle.**
What is the length of the rectangle?

10 ✓ ✓

What is the width of the rectangle?

8 ✓ ✓

Student B does a nice job of working with the inverse relationships in part 3 and explaining how to use the formula to find the side if given the number of inside dots. Student B is able to generate a formula for part 5 which will for all rectangles, but can't apply to finding the dimensions of the rectangle when given the interior dots.. The student can't use the formula to find the dimensions of a rectangle with 63 inside dots. The student has noticed another pattern. In the examples in the table, all interior dots are even numbers. Sometimes students try to generalize about things that are not true for all cases.

S	1^2	2^2	3^2	4^2	5^2	6^2
P	4	8	12	16	20	24
I	0	1	4	9	16	25

1. Fill in the empty boxes in Sally's table.

2. Write a rule or a formula for finding the number of dots inside a square when you know the length of a side of the square.

$(S-1)^2 = I$ (Side length -1) squared equals the # of dots inside the square

3. There are 49 dots inside a square. What is the length of one side of the square? Explain your reasoning.

8 is the side length because the side length before squared equals the # of dots inside the side length after it. (Ex. S 1 1^2 1 3 4 I) OR $(8-1)^2 = 4$

L (in squares)	2	3	4	5	6
W (in squares)	1	2	3	4	5
I	0	2	6	12	20

All even

4. Fill in the empty boxes in the table above.

5. Write a rule or formula for finding the number of dots inside a rectangle (I) when you know the length (L) and the width (W) of the rectangle.

$(L-1)(W-1) = I$ (Length -1) × (Width - 1) = the number of dots inside the rectangle.

6. There are 63 dots inside a rectangle.
What is the length of the rectangle?

What is the width of the rectangle?

can't do it because is not an even #

(9)

Student C has a similar problem to Student B. The student is not able to use the formula to find the dimensions of a rectangle with 63 inside dots. The student focuses on the numbers in the table rather than thinking about the properties in the geometry of the pattern to find out what will hold true for all cases.

Student C

L (in squares)	2	3	4	5	6
W (in squares)	1	2	3	4	5
I	0	2	6	12 ✓	20 ✓

4. Fill in the empty boxes in the table above.

5. Write a rule or formula for finding the number of dots inside a rectangle (I) when you know the length (L) and the width (W) of the rectangle.

$(L-1)(W-1) = I$ ✓ ✓

6. There are 63 dots inside a rectangle.
What is the length of the rectangle?

What is the width of the rectangle?

$9 \cdot 7 = 63$

It's not possible because it is not an even number. And it doesn't have two consecutive factor

Student D has found a formula that only works for one case of rectangles. The formula will only work for those rectangles whose length is one unit longer than the width. Therefore Student D cannot use the formula to help find the dimensions of a rectangle with 63 interior dots.

Student D

L (in squares)	2	3	4	5	6
W (in squares)	1	2	3	4	5
I	0	2	6	12	20

4. Fill in the empty boxes in the table above.

5. Write a rule or formula for finding the number of dots inside a rectangle (I) when you know the length (L) and the width (W) of the rectangle.

(L-2)(W-1)+(L-2)

6. There are 63 dots inside a rectangle.
What is the length of the rectangle?

What is the width of the rectangle?

Student E has also found a formula that works only for rectangles where the length is one more than the width. While the generalization is incorrect (the task was looking for a formula for all cases of rectangles), the student was able to use the formula to find numbers to fit the pattern in part 6. However the numbers will not map back to the geometric situation.

Student E

L (in squares)	2	3	4	5	6
W (in squares)	1	2	3	4	5
I	0	2	6	12	20

4. Fill in the empty boxes in the table above.

5. Write a rule or formula for finding the number of dots inside a rectangle (I) when you know the length (L) and the width (W) of the rectangle.

$W(L-2)=I$

$3(4-2)=6$ ✓

$6(7-2)=30$ ✓

6. There are 63 dots inside a rectangle.
What is the length of the rectangle?

What is the width of the rectangle?

$7(11-2)=$

$7(9)=63$

$63=(L-2)W$

$9-2=7\cdot 9=63$

Student F makes a common mistake of finding a recursive relationship of adding the next higher odd or even number each time. This is a cumbersome relationship to use because it requires generating the entire list to solve for a particular solution.

Student F

S	1	2	3	4	5	6
P	4	8	12	16	20	24
I	0	1	4	9	16	25

1. Fill in the empty boxes in Sally's table.

2. Write a rule or a formula for finding the number of dots inside a square when you know the length of a side of the square.

For P add 4 and For I you have to add the next odd number.

3. There are 49 dots inside a square. What is the length of one side of the square? Explain your reasoning.

The length of it is 7 because I add the next odd number till I got 49.

25 1 13 36 49

and the number of dots inside (I).

L (in squares)	2	3	4	5	6
W (in squares)	1	2	3	4	5
I	0	2	6	12	20

4. Fill in the empty boxes in the table above.

5. Write a rule or formula for finding the number of dots inside a rectangle (I) when you know the length (L) and the width (W) of the rectangle.

You have to add the next area number.

6. There are 63 dots inside a rectangle.
What is the length of the rectangle? 15 x x

What is the width of the rectangle? 15 x x

20 30 42 56 72

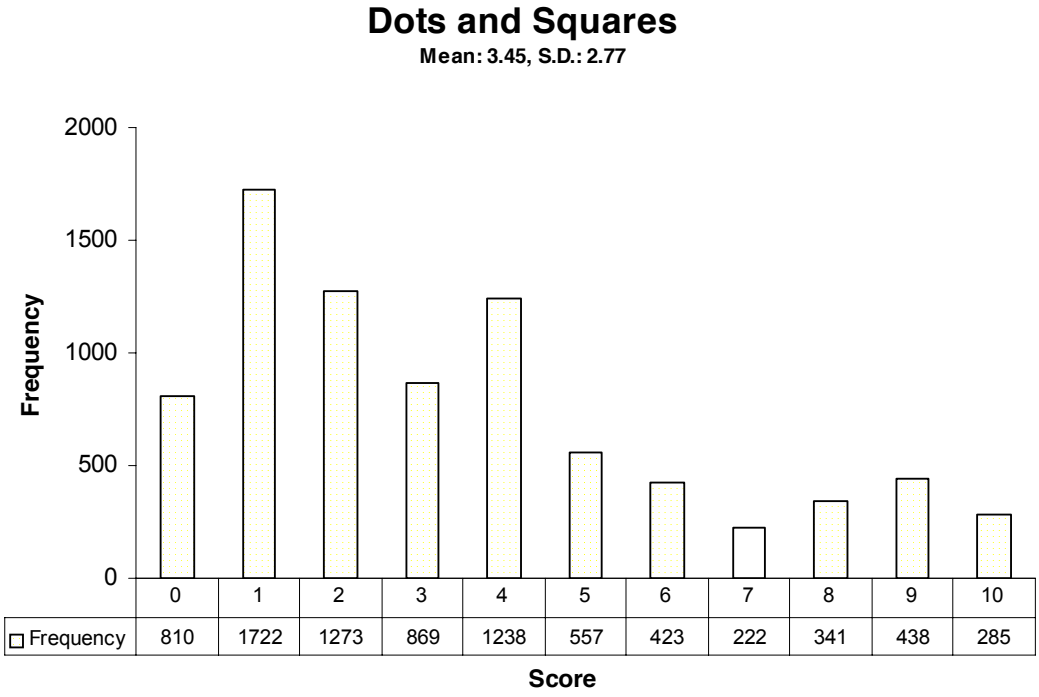
56 + 14 = 72

92 + 14 = 156 + 14 = 72

4

4

10



Score:	0	1	2	3	4	5	6	7	8	9	10
% < =	9.9%	31.0%	46.5%	57.2%	72.3%	79.1%	84.3%	87.0%	91.2%	96.5%	100.0%
% > =	100.0%	90.1%	69.0%	53.5%	42.8%	27.7%	20.9%	42.8%	13.0%	8.8%	3.5%

The maximum score available for this task is 10 points.
The cut score for a level 3 response is 6 points.

Most students (about 90%) could fill in the table with the correct perimeter for each square. Many students (about 70%) could also find the number of inside dots for a square. A little less than half the students (43%) could find the perimeter and the inside dots for the squares and the rectangles. About 21% could meet standards by filling in the tables for perimeter and dots and find the dimensions of square with 49 inside dots. Less than 5% of the students met all the demands of the task. Almost 10% of the students scored no points on this task. Of those more than 60% attempted some part of the task.

Dots and Squares

Points	Understandings	Misunderstandings
0	10% of the students scored no points. Almost 64% of them attempted the problem.	About 28% of the students did not attempt this task or the final task on the test. Time may or may not have been an issue.
1	Students could fill in the table for the perimeter values for the square.	Students did not count inside dots, but tried to find numerical patterns like going up by 3 or 4 every time, doubling, or having the inside always 4 less than the perimeter. They looked at only a couple of numbers in the table to find their rule, instead of testing the rule for all values in the table.
2	Students could fill in the table for perimeter and inside dots for squares.	Most students would not attempt any type of rule or generalization. Of the students who missed part 2, 33% did not attempt it. Of the students who missed part 5, 41% did not attempt it.
4	Students could fill in the tables for squares and rectangles.	About 15% of the students found a recursive relationship for the inside dots, like adding the next higher odd number. 11% wanted to count up by 3 or 4 every time. 7% wanted to multiply the number of sides by 4 to find the inside dots.
6	Students could fill in both tables and find the dimensions of a square if they knew the number of inside dots.	Many students used counting or drawing strategies. They could not find a rule or formula.
8	Students could fill in both tables, find a rule for inside dots in a square use the rule to find the side length if they knew the number of inside dots.	
9	Students could fill in the tables, find a rule for inside dots in squares and rectangles, and use their rule to find the dimensions of a square given the number of inside dots.	Students may have found a rule that only works for certain cases of rectangles and so their rule would not help them with part 6. They may have paid attention to more than one possible pattern in the rectangles which limited their thinking in part 6. See the work of Students B and C.
10	Students could make generalizations about geometric patterns to predict the number of inside dots and use the pattern to work backwards from inside dots to dimensions.	

Based on teacher observations, this is what eighth grade students seemed to know and be able to do:

- Find the perimeter of squares and rectangles.
- Find the number of inside dots for a square or rectangle.

Areas of difficulty for eighth graders, eighth grade students struggled with:

- Writing rules or formulas for geometric patterns.
- Using rules to work backwards.
- Understanding how to check a rule to see if it works for all the cases in the given information. (Making generalizations on too little information.)

Questions for Reflection on Dots and Squares:

- What types of experiences or problems have your students had with making rules or formulas to match geometric patterns?
- When working with patterns, have the problems focused only on linear patterns?
- What questions or experiences do you ask students to help them see the relationships between variables instead of looking at how patterns grow (finding recursive relationships)?

Look at student responses to part 2. How many of your students:

Did not attempt a rule	Gave a counting or drawing strategy	Goes up by an increasing odd number	Goes up by 4 (or 3) every time	Multiply the sides by 4	Looked at interior instead of exterior so rules like: SxS or LxL or LxW

Look at student responses to part 5. How many of your students:

Did not attempt a rule	Gave a counting or drawing strategy	Goes up by increasing even number	Goes up by 4 every time	L x W	Gave rule that only works for certain rectangles

In part 5 many students gave rules that would only work for rectangles where the length was one unit longer than the width. What are some of the formulas? Make a list.

Why won't these formulas work for all cases of rectangles? How do you know? What would students have needed to focus on in the drawings or tables to know these rules wouldn't apply? What experiences have students had looking at different cases to make a proof? What types of problems have students worked on that required justifications?

- Have students in your class had opportunities to do investigations on their own and try to make generalizations from the data?

Teacher Notes:

Instructional Implications:

When students look at pattern problems, it is helpful to visualize what is changing and what is staying the same. As they progress through the grades this information could be used to help them write a rule or formula. At this grade level, they can no longer rely on drawing pictures or doing repeated addition to find the solutions to complex problems. Students at this grade level should be proficient at answering a variety of questions about patterns. They need to recognize that patterns can grow in more than one direction and be able to investigate those changes. Students should work with patterns with exponential growth as well as linear growth. Students should develop the habit of verifying their rules or formulas to see if they work for more than one example.

8th grade Task 5 Number Pairs

Student Task	Identify number pairs on a coordinate grid.
Core Idea 3 Algebra and Functions	**Understand relations and functions, analyze mathematical situations, and use models to solve problems involving quantity and change.** • Explore relationships between symbolic expressions and graphs of lines • Relate and compare different forms of representations for relationships including words, tables, graphs in the coordinate plane and symbols (7th grade)