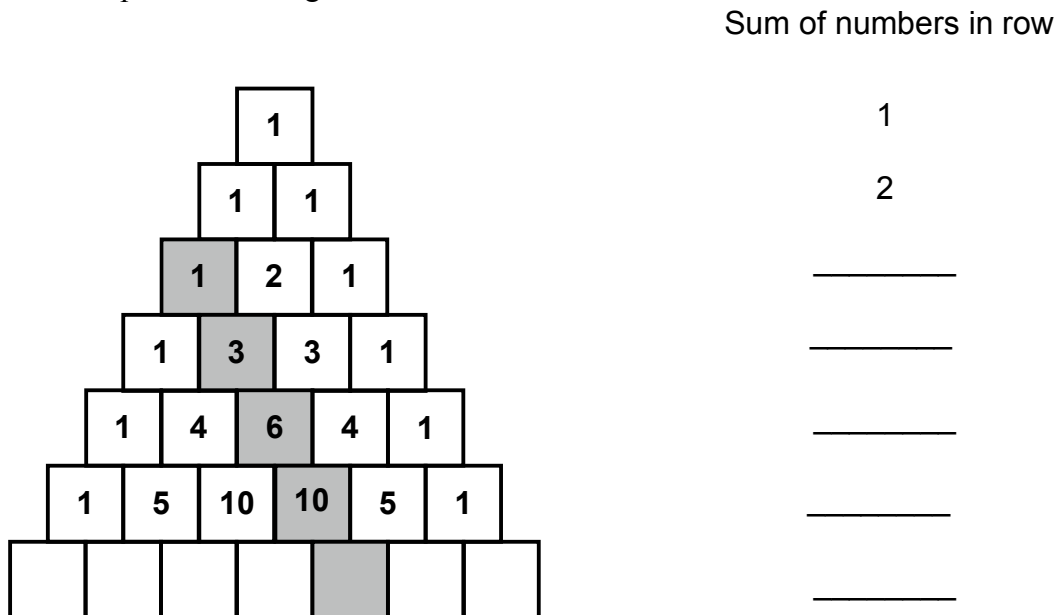


A Number Pattern

This problem gives you the chance to:

- describe and extend a numeric pattern

This is a number pattern. It can go on and on.



1. Which numbers appear just once in the part of the pattern that is shown above?

2. In this pattern, each row begins and ends with the number 1.
The other numbers are the sum of the two numbers above it.
For example, $10 = 6 + 4$.

Continue the pattern, by writing numbers in the row of empty squares.

3. a. Find the sum of the numbers in each of the rows. The first two have been done for you.
Write your answers on the diagram above.

b. What do you notice about the sequence of numbers in the Totals column?

4. Look at the numbers that have been shaded.

What do you notice about the sequence of numbers that have been shaded?

A Number Pattern	Rubric	
<ul style="list-style-type: none"> • • The core elements of performance required by this task are: • • describe and extend a numeric pattern • <p>Based on these, credit for specific aspects of performance should be assigned as follows</p>	points	section points
1. Gives correct answer: 2 Accept 6 and 20	1	1
2. Gives correct answers: 1, 6, 15, 20, 15, 6, 1 <i>Partial credit</i> One error	2 (1)	 2
3.a. Gives correct answer: 1, 2, 4, 8, 16, 32, 64 <i>Partial credit</i> One error b. Gives correct answer such as: The numbers double each time. or Powers of 2	2 (1) 1	 3
4. Gives correct answer such as: The difference between consecutive numbers (2, 3, 4, 5) increases by one each time. <i>Partial credit</i> For one error or an incomplete statement.	2 (1)	 2
Total Points		8

A Number Pattern

Work the task and look at the rubric. What are the big mathematical ideas in this task?

Now look at student work for part 1. How many of your students thought the answer was 2? _____ How many just put 6? _____ How many put 1? _____ How many put 10? _____ Other answers: _____ What do you think caused students difficulty about this part of the task? What might they have been thinking?

Look at student work for part 2, filling in the bottom row of the table. How many students put:

- Correct pattern: 1,6,10,15,20,15,10, 6,1?
- Pattern without 1/s on both or either end?
- Pattern of increasing or decreasing numbers: example: 1,2,6,10,15,21,28?
- Patterns that weren't symmetrical?

What other types of misconceptions did you notice in student errors?

In part 3a students are asked to add the numbers in the rows. Did your students:

- Give the correct numbers?
- Make one calculation error in the final number?
- Make a list of counting numbers: 1,2,3,4,...?
- Come up with a pattern rather than adding numbers?

What don't students understand? What clues are they not paying attention to?

In part 3b students are asked to describe the pattern of the totals.

- Could your students describe the pattern of doubling or multiplying by 2?
- Did they give a vague answer, like it's a pattern, they're in order, or they get bigger?
- Did they talk about odd and/or even relationships?
- Did they describe a different pattern in the figure, like every row begins and ends in 1?
- Give a non-mathematical answer, such as the numbers are going down \Downarrow ?

How do we help students develop the ability to look for more simple patterns, like odd and even? How do we help students express more details about what they're thinking rather than stop short of a full description?

In part 4 students are asked to describe the pattern of the shaded numbers.

- How many of your students could describe the pattern? _____ Partially describe the pattern?
- How many described a different pattern in the figure, such as every row begins and ends with one? To get the bottom number, add the two numbers above?
- How many gave vague answers, such as it's a sum, a pattern, or its skipping numbers?
- How many discussed something about odd and even?
- Just noticed that $1 + 3 + 4 =$ the final number 10?

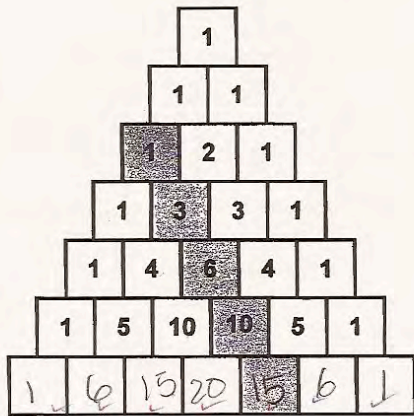
What types of responses did you like? What qualities made them different?

Looking at Student Work on A Number Pattern

Student A is able to look at all the numbers in the figure to find the numbers that appear only once. The student can make the generalization for the numbers in the sum column and give a description of the progression of numbers in the shaded row and how they are formed.

Student A

This is a number pattern. It can go on and on.



Sum of numbers in row

1
2
4 ✓
8 ✓
16 ✓
32 ✓
64 ✓

1. Which numbers appear just once in the part of the pattern that is shown above?

2 and 20 ✓ | |

2. In this pattern, each row begins and ends with the number 1. The other numbers are the sum of the two numbers above it. For example, $10 = 6 + 4$.

Continue the pattern, by writing numbers in the row of empty squares.

2 2

3. a. Find the sum of the numbers in each of the rows. The first two have been done for you. Write your answers on the diagram above.

2 2

b. What do you notice about the sequence of numbers in the Totals column?

I noticed that each number is the product of the number above it times 2.

1
x 2
2
x 2
4
x 2
8
x 2
16
x 2
32
x 2
64

4. Look at the numbers that have been shaded.

What do you notice about the sequence of numbers that have been shaded?

I noticed that each number except 1 is the sum of that number plus the numbers in numerical order.
 $1 + 2 = 3 + 3 = 6 + 4 = 10 + 5 = 15$

8

Student B notices the 6 but doesn't notice that the 2 also appears only once. The student gives only a vague description in part 3b and is just difficult to understand in part 4. While "getting bigger" might be an interesting observation at primary grades, it is not at the level of observation expected for a sixth grader. How do we help students get progressively deeper in their thinking and quality of responses as they move through the grades?

Student B

This is a number pattern. It can go on and on.

Sum of numbers in row

1	
2	
4	✓
8	✓✓
16	✓
32	✓
64	✓

Handwritten notes:
 $35 + 10 = 45$
 $45 + 10 = 55$
 $55 + 2 = 57$

1. Which numbers appear just once in the part of the pattern that is shown above?

$6^x \wedge$

00
2. In this pattern, each row begins and ends with the number 1. The other numbers are the sum of the two numbers above it. For example, $10 = 6 + 4$.

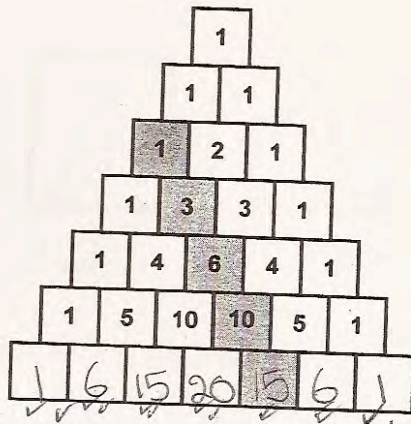
 Continue the pattern, by writing numbers in the row of empty squares. 2 2
3. a. Find the sum of the numbers in each of the rows. The first two have been done for you. Write your answers on the diagram above. 2 2
 b. What do you notice about the sequence of numbers in the Totals column?
That the number is getting bigger and bigger 0 0
4. Look at the numbers that have been shaded. What do you notice about the sequence of numbers that have been shaded?
I notice that they got a different way to get 15! 0 0

8

Like many students, Student C does not look at the entire figure and misses that “2” is only used once. In 3b the student makes an observation about the structure of the figure rather than answering the question about the totals column. In part 4 the student is vague about which two numbers are added to equal the shaded pattern.

Student C

This is a number pattern. It can go on and on.



Sum of numbers in row

1
2
4 ✓
8 ✓
16 ✓
32 ✓
64 ✓

1. Which numbers appear just once in the part of the pattern that is shown above?

20 ✗ 00

2. In this pattern, each row begins and ends with the number 1. The other numbers are the sum of the two numbers above it. For example, $10 = 6 + 4$.

Continue the pattern, by writing numbers in the row of empty squares.

2 2

3. a. Find the sum of the numbers in each of the rows. The first two have been done for you. Write your answers on the diagram above.

2 2

b. What do you notice about the sequence of numbers in the Totals column?

I notice that each row has a 1 at the end and at the begin.

4. Look at the numbers that have been shaded. What do you notice about the sequence of numbers that have been shaded?

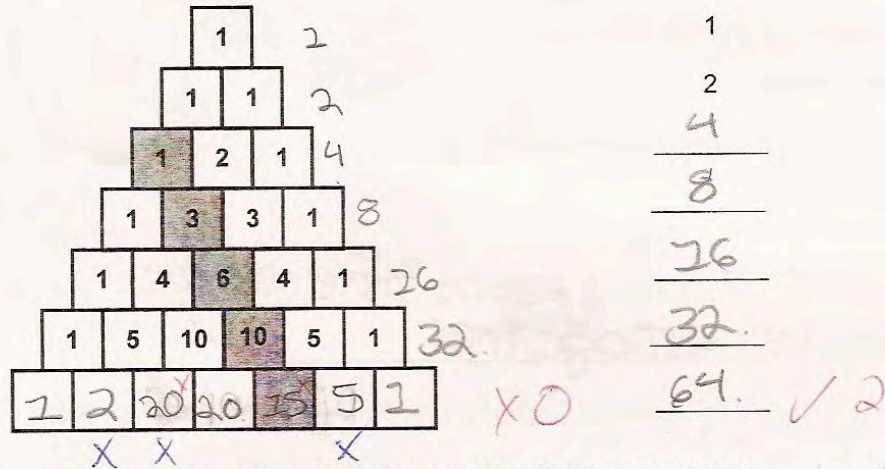
I notice that two numbers are equal to the numbers shaded.

Student D gives a similar response to Student C for part 4. In filling out the bottom row, the student has failed to notice that the numbers in each row are symmetrical or that the middle or middle 2 numbers are the highest numbers in the row. *Do we give students ample opportunities to describe in detail rich patterns and all the things they notice? How do we help students start to piece together what are the relevant features in the structure being described?*

Student D

This is a number pattern. It can go on and on.

Sum of numbers in row



1. Which numbers appear just once in the part of the pattern that is shown above?

2 ✓ ✓ 1

2. In this pattern, each row begins and ends with the number 1. The other numbers are the sum of the two numbers above it. For example, $10 = 6 + 4$.

Continue the pattern, by writing numbers in the row of empty squares.

X 0

3. a. Find the sum of the numbers in each of the rows. The first two have been done for you. Write your answers on the diagram above.

✓ 2

b. What do you notice about the sequence of numbers in the Totals column?

The numbers are being doubled.

✓ ✓ 1 1

4. Look at the numbers that have been shaded.

What do you notice about the sequence of numbers that have been shaded?

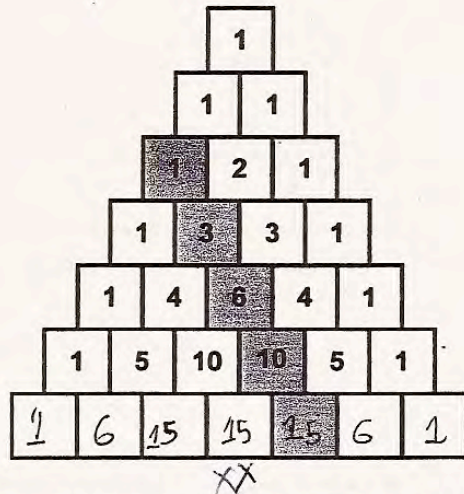
They are the sum of numbers above it.

X 0

Student E is able to list most of the numbers in the totals row. The student seems so used to working with patterns and tables that the student inserts a figure number to the left of the total. However, the student's pattern descriptions are very vague. The student may notice the patterns, but doesn't have facility in being specific. *How do we communicate to students the qualities we want in a good response? How do we give feedback about not giving enough details so that students learn to write better mathematically?*

Student E

This is a number pattern. It can go on and on.



Sum of numbers in row

1 ✓
 2 ✓
 3 = 4 ✓ ✓
 4 = 8 ✓ ✓
 5 = 16 ✓ ✓
 6 = 32 ✓ ✓
 7 = 49 ✗ ✗

1. Which numbers appear just once in the part of the pattern that is shown above?

2 ✓ ✓

2. In this pattern, each row begins and ends with the number 1. The other numbers are the sum of the two numbers above it. For example, $10 = 6 + 4$.

Continue the pattern, by writing numbers in the row of empty squares.

3. a. Find the sum of the numbers in each of the rows. The first two have been done for you. Write your answers on the diagram above.

b. What do you notice about the sequence of numbers in the Totals column?

they keep getting bigger ✗ ✗

4. Look at the numbers that have been shaded.

What do you notice about the sequence of numbers that have been shaded?

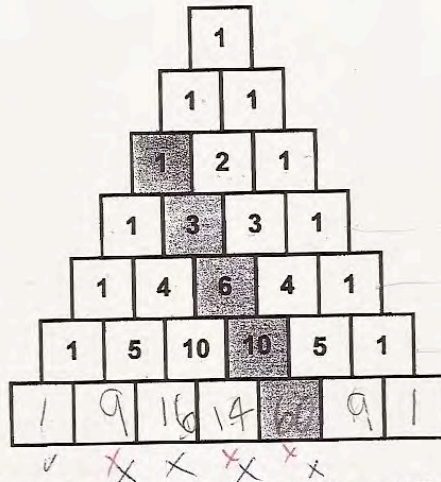
they are added numbers ✗ ✗

Look at the work of Student F. *How has this student interpreted the instructions “add the two numbers above”?* Notice that the student is looking at discrete digits when trying to think about the totals column rather than thinking about the number, thirty-two versus a three and a two. *What types of experience does this student need?* Also the student sees 3 and 6 and thinks about skipping 3’s. *What are some of the errors in this thinking? Why do you think the student got confused?*

Student F

This is a number pattern. It can go on and on.

Sum of numbers in row



1
2
4
8
16
32 ✓
42 x x

1. Which numbers appear just once in the part of the pattern that is shown above?

2 ✓

2. In this pattern, each row begins and ends with the number 1. The other numbers are the sum of the two numbers above it. For example, $10 = 6 + 4$.

Continue the pattern, by writing numbers in the row of empty squares.

3. a. Find the sum of the numbers in each of the rows. The first two have been done for you. Write your answers on the diagram above.

b. What do you notice about the sequence of numbers in the Totals column?

That except for the 2 ones they are all different.

4. Look at the numbers that have been shaded.

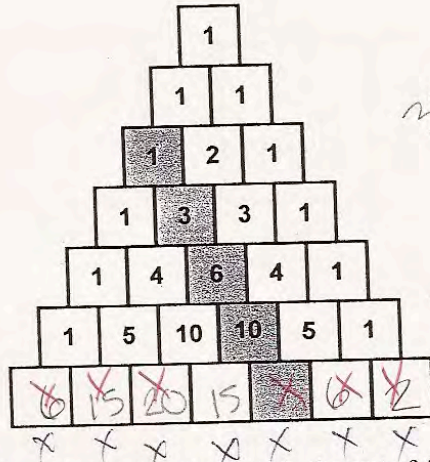
What do you notice about the sequence of numbers that have been shaded?

That the first 3 are skipping 3 x x

Student G also has difficulty interpreting “add the two numbers above”. *What error has this student made?* The student didn’t understand what was expected in the sums column, but has worked out the totals in the empty space. The student has actually described the correct pattern, but not for the numbers appearing in his own sums column.

Student G

This is a number pattern. It can go on and on.



Sum of numbers in row

1
2
3
4
5
6
10

1. Which numbers appear just once in the part of the pattern that is shown above?

2 ✓ 1 1

2. In this pattern, each row begins and ends with the number 1. The other numbers are the sum of the two numbers above it. For example, $10 = 6 + 4$.

0 0

Continue the pattern, by writing numbers in the row of empty squares.

0 0

3. a. Find the sum of the numbers in each of the rows. The first two have been done for you. Write your answers on the diagram above.

b. What do you notice about the sequence of numbers in the Totals column?

If you total one column the column above will be half of that.

4. Look at the numbers that have been shaded.

What do you notice about the sequence of numbers that have been shaded?

The difference between each number get bigger by 2 like 1 and 3 a diff. of 2 3 and 6 diff 3 and 10 diff. of 4.

8 2 3

Student H doesn't look at enough of the diagram to find the pattern for the bottom row. The student seems to look at the 5's and 10's in the row above and then put multiples. The student has noticed a pattern that the 2nd number in each row is one more than the previous and that the rows are symmetrical. The student is still struggling with basic computation, so he doesn't see the pattern of doubling within the rows. Notice that the student attempts to total the totals.

Student H

This is a number pattern. It can go on and on.

- Which numbers appear just once in the part of the pattern that is shown above?
- In this pattern, each row begins and ends with the number 1. The other numbers are the sum of the two numbers above it. For example, $10 = 6 + 4$.
Continue the pattern, by writing numbers in the row of empty squares.
- a. Find the sum of the numbers in each of the rows. The first two have been done for you. Write your answers on the diagram above.
b. What do you notice about the sequence of numbers in the Totals column?

- Look at the numbers that have been shaded. What do you notice about the sequence of numbers that have been shaded?

Student Task	Describe and extend a number pattern.
Core Idea 3 Algebra and Functions	<p>Understand relations and functions, analyze mathematical situations, and use models to solve problems involving quantity and change.</p> <ul style="list-style-type: none"> • Represent, analyze, and generalize a variety of relations and functions with tables, graphs, and words.
Core Idea 1 Number and Operation	<ul style="list-style-type: none"> • Describe classes of numbers according to characteristics such as the nature of their factors.

Mathematics of this task:

- Identifying a variety of number patterns and being able to describe the patterns mathematically and completely
- Looking at a pattern with many different structural elements and identifying the relevant features
- Making connections between structural elements in a pattern

Based on teacher observation, this is what sixth graders know and are able to do:

- Add the totals of the rows
- Notice and describe a doubling pattern
- Continuing the pattern for the first two and last two numbers in the bottom row

Areas of difficulty for sixth graders:

- Finding the pattern for the middle numbers in the bottom row
- Describing the pattern for the shaded numbers
- Looking through the entire pattern to see that the 2 is only used once

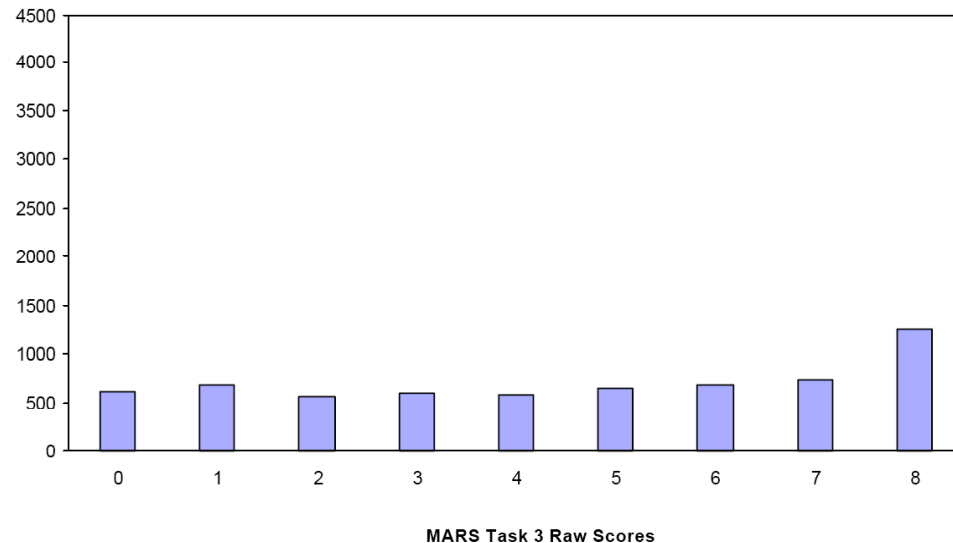
Task 3 - Number Pattern

Mean: 4.47 StdDev: 2.73

Table 32: Frequency Distribution of MARS Test Task 3, Grade 6

Task 3 Scores	Student Count	% at or below	% at or above
0	616	9.7%	100.0%
1	683	20.4%	90.3%
2	565	29.3%	79.6%
3	599	38.7%	70.7%
4	584	47.9%	61.3%
5	638	57.9%	52.1%
6	685	68.7%	42.1%
7	733	80.2%	31.3%
8	1256	100.0%	19.8%

Figure 41: Bar Graph of MARS Test Task 3 Raw Scores, Grade 6



The maximum score available for this task is 8 points.

The minimum score needed for a level 3 response, meeting standards, is 5 points.

Most students, 90%, could notice that the 2 is only used once or do most of the totals row. Many students 80% could do both. More than half the students, 52%, could fill in the bottom row, give most of the totals, and describe the doubling pattern in the totals. Some students, about 40%, could also note that the 2 is only used once. Almost 20% of the students could meet all the demands of the task, including describing how the shaded numbers are formed by adding a progression of consecutive numbers, starting with 2. Almost 10% of the students scored no points on the task. 85% of the students with this score attempted the task.

A Number Pattern

Points	Understandings	Misunderstandings
0	85% of the students with this score attempted the task.	Students had difficulty identifying the number that was used only once. 21% of the students thought it was just the 6. 4% thought it was 1.
1	Students either knew that “2” was only used once or they can fill in most of the bottom row.	Some students did not notice that each row starts and ends with one, 3%. Some students did not notice the pattern was symmetrical, 19%. Some students did not notice the middle number(s) are largest, 20%.
2	Students noted 2 was used only once and could fill in most of the bottom row.	Students had difficulty with the totals column. 13% put 1,2,3... 9% just had an incorrect answer for the final sum. 5% did some other pattern.
5	Students could fill in the bottom row, add the totals, and notice the doubling pattern.	Some students still had difficulty with noticing that 2 was only used once.
6	Students could do all of the task except part 4, the pattern of the shaded numbers.	Almost 8% of the students did not attempt this part of the task. 8% might have understood the pattern, but writing was too unclear. 6% noticed that $1+3+4=10$. 4% gave vague answers, such as its getting bigger.
8	Students could work with a variety of patterns by describing them or extending them, find totals, and look at all the clues in the pattern to see that 2 is only used once.	

Implications for Instruction

Students need to be able to find and extend patterns. At this grade level they should be exposed to a variety of patterns, beyond looking for odd and even numbers or growing or decreasing by a set amount. They should recognize doubling patterns or growing by an increasing number. Students need to be pushed to describe what they see in more detail. Students often try to reach for some minimum level of explanation. This habit of mind prevents them looking at numbers closely enough to find other patterns.

Giving students feedback on the quality of their explanations and examples of types of explanations that are possible helps to raise the bar on student thinking. An important instructional question to think about at this grade level is how to increase the cognitive demand from expectations at previous grade levels.

Ideas for Action Research – Using Tasks for Instruction and the Importance of Feedback

Most of the time MARS tasks are used for assessment purposes. But in following years, consider using them for instructional purposes. At a recent lesson study, a group of teachers asked the question about how they could improve the quality of answers given by students. They didn't feel that students challenged themselves to think deeply enough. Teachers were also concerned about how to give feedback to students when their class sizes were so large. They didn't feel they had the time to write notes to every student, yet they knew from articles, such as "Inside the Black Box" by Black and Wilam, that specific feedback is one of the foremost factors in furthering student learning.

Consider how this task might be used as a whole class learning activity to work on this issue.

You might start by giving pairs of students just the diagram. Ask them to find and describe as many patterns in the diagram as they can. Have the pairs glue their diagram in the middle of a large piece of poster paper and then write out their patterns, using colored markers to help highlight what they are describing.

Students might then share out in groups of 3 to 5 pairs, with students asking each other clarifying questions.

After everyone has had a chance to explore the patterns and make sense of the context, think about asking a re-engagement question to push their thinking. For example:

Margie says, "I think this might be like other pattern problems. I bet the teacher will want to know how to predict future numbers. What patterns will help us know what comes in the next rows?"

See if this stimulates students to find new or different patterns. Now try another push.

Ford says, "I also bet the teacher will ask how we know if our predictions are correct." Kristi adds, "I think that if we add the total for each row it might make a pattern that can help us."

Do you think Kristi is correct? Why or why not?

Finally push students to evaluate responses and give their own feedback. It is important for them to develop their own internal logic about what makes a detailed explanation. Give them the rest of the task then pose a question, such as:

I noticed some patterns from other classes. Look at part 3b. What do you think each student is thinking about?

The number is getting bigger and bigger.

Each row has a "1" at the end and the beginning.

Each number is the product of the number above it times 2.

Except for the 2 ones they are all different.

What do you like about their patterns? How might their explanations be improved?

Now look at part 4. Which responses do you like the best and why?

They are the sum of the numbers above it.

That the first three are skipping by 3's.

I noticed that each number except 1 is the sum of the number plus the numbers in numerical order. So, $1 + 2 = 3$, $3 + 3 = 6$, $6 + 4 = 10$, $10 + 5 = \dots$

The difference between each number gets bigger by one like 1 and 3 are a difference of 2, 3 and 6 are a difference of 3, . . .

They are added numbers.

How might you improve these explanations?

How does this lesson help all students follow the mathematics of the task? How does this lesson help push students to think about the qualities of a good explanation?