

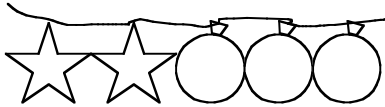
Birthday Decoration

This problem gives you the chance to:

- find and extend a pattern

Cameron is decorating the house to celebrate his mom’s birthday. He makes a pattern with silver stars and red balloons.

The first piece looks like this.



When he adds piece #2 it looks like this.



1. Draw piece #3 onto the pattern above.

Cameron needs to know how many stars and balloons he will be using. He makes this table to help.

number of pieces	1	2	3	4	5
number of stars	2	4			
number of balloons	3	6			

2. Fill in the numbers in the table for three pieces.
3. How many stars and balloons will he need to make five pieces?
Write your answer in the table above.
4. When he started decorating, Cameron had 26 balloons and 19 stars. He said that he would be able to make nine pieces. Is he correct? _____
Explain your work.

Birthday Decoration	Rubric	
<ul style="list-style-type: none"> • • The core elements of performance required by this task are: • • find and extend a pattern • <p>Based on these, credit for specific aspects of performance should be assigned as follows</p>	points	section points
1. Draws the 2 stars and 3 balloons to extend the pattern	2 x 1	2
2. Fills in the table correctly: 6 stars and 9 balloons	2 x 1	2
3 Fills in the table correctly: 10 stars and 15 balloons	2 x 1	2
<p>4 Gives correct answer: No and explanation such as: He has 19 stars which would make 9 pieces of the pattern with one star left over. There are 26 balloons: 26 divided by 3 is 8 with a remainder of 2. So Cameron can only make 8 pieces of the pattern</p> <p><i>Partial credit</i> For a partially correct answer.</p>	<p>2</p> <p>(1)</p>	2
Total Points		8

Birthday Decoration

Work the task. What are the big mathematical ideas? _____

How often do students in your class work problems in context?

What types of opportunities do they have to interpret the answers to their calculations?

Look at student work in part 1, extending the pattern. How many of your students:

Made a correct drawing	Made no drawing	Forgot to draw the balloons	Made only 2 balloons	Other

How do we help students notice all the attributes of a pattern? Do you ask students questions, like what are the elements in the pattern? How many of each kind? What does the first pattern piece look like?

Now look at student work for part 4. How many of your students:

- Gave a complete explanation? _____
- Thought there were enough pieces? _____
- Thought about individual pieces rather than the whole pattern of 5 shapes? _____
- Added the $19 + 26 = 45$ and $9 \times 5 = 45$, so there are enough pieces? (*Didn't distinguish between the types of pieces?*) _____
- Thought 19 is not an even number, so it won't work? _____
- Extended the table but couldn't interpret the answer? _____
- Other? _____

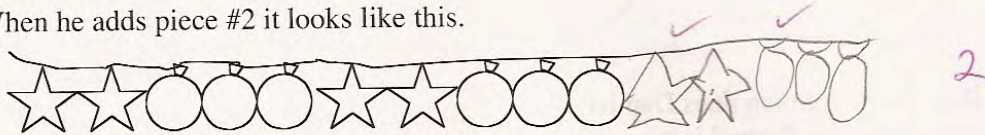
Why do you think students struggled with the explanations? How do you craft lessons to help students develop the logic of making convincing arguments or explanations? How might you use some of their thinking to plan a short re-engagement lesson on part 4?

Looking at Student Work on Birthday Decorations

Student A is able to draw the pattern and extend the pattern using a table. *What are the qualities in the argument that make it convincing?*

Student A

When he adds piece #2 it looks like this.



1. Draw piece #3 onto the pattern above.

Cameron needs to know how many stars and balloons he will be using. He makes this table to help.

number of pieces	1	2	3	4	5	6	7	8	9
number of stars	2	4	6 [✓]	8	10 [✓]	12	14	16	18
number of balloons	3	6	9 [✓]	12	15 [✓]	18	21	24	27

2. Fill in the numbers in the table for three pieces.

3. How many stars and balloons will he need to make five pieces?
Write your answer in the table above.

4. When he started decorating, Cameron had 26 balloons and 19 stars. He said that he would be able to make nine pieces. Is he correct? NO
Explain your work.

Cameron is not able to make the ninth piece because 8
he needs eighteen stars and he has nineteen that's ok but he needs 27 balloons and he has 26. So he is not able to.

Student B also makes a convincing explanation for part 4. *What are the qualities in this argument that make it convincing?*

Student B

4. When he started decorating, Cameron had 26 balloons and 19 stars. He said that he would be able to make nine pieces. Is he correct? no ✓
 Explain your work.

I counted by 2's and 3's until I reached 2×9 and 3×9 . For 2's I had enough. But for 3's I came up with 27 which is more than 26 so I knew it wouldn't work.

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Page 2

Birthday Decoration

8

8

Student C does a lot of correct calculations in part 4. *What is missing to make this a convincing argument?*

Student C

number of pieces	1	2	3	4	5
number of stars	2	4	6	8	10
number of balloons	3	6	9	12	15

counted by 2's
 count by 3's

2. Fill in the numbers in the table for three pieces.

3. How many stars and balloons will he need to make five pieces?
 Write your answer in the table above.

10
 +15
 25

4. When he started decorating, Cameron had 26 balloons and 19 stars. He said that he would be able to make nine pieces. Is he correct? no
 Explain your work.

First, I divided $26 \div 3$ which equaled 8 remainder of 2. Then, I divided $19 \div 2$ which equaled 9 remainder of 1.

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Page 2

Birthday Decoration

8

$2 \overline{) 19} \quad 9 \text{ R.1}$
 $3 \overline{) 26} \quad 8 \text{ R.2}$

Student D also does calculations, but does not know how to interpret the solutions in terms of the context. *What is important about comparing the 18 and the 19? If 3×9 is not 26, why is this important or not important in the context of decorations? What questions might you pose to the class using this piece of student work?*

Student D

4. When he started decorating, Cameron had 26 balloons and 19 stars. He said that he would be able to make nine pieces. Is he correct? no
Explain your work. (1)

$2 \times 9 = 18$ not 19 and 3×9 is not 26.

Look at the work of Student E. *Why might he think there are enough balloons and stars?*

Student E

4. When he started decorating, Cameron had 26 balloons and 19 stars. He said that he would be able to make nine pieces. Is he correct? ✓
Explain your work. x 0

He has enough balloons for nine pairs and stars also

8 (6)

Now look at the work of Student F. He seems to have lost track of the pattern and is thinking about individual pieces.

Student F

4. When he started decorating, Cameron had 26 balloons and 19 stars. He said that he would be able to make nine pieces. Is he correct? NO
Explain your work.

He has more than nine pieces. x 0

Student G has done some thinking and has a detailed explanation. *What is missing to make the explanation complete?*

Student G

number of balloons	3	6	9	12	15
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2. Fill in the numbers in the table for three pieces. 2

3. How many stars and balloons will he need to make five pieces?
Write your answer in the table above. 2

4. When he started decorating, Cameron had 26 balloons and 19 stars. He said that he would be able to make nine pieces. Is he correct? NO
Explain your work.

If you keep counting for 3's it won't be correct because 26 is not in the pattern for 3's. On the chart for the stars it goes up by twos for the balloons on the chart it goes up by threes. If you keep counting for 2's it won't be correct because its not an even number.

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Like Student G, Student H is sidetracked by the pattern, forgetting about the relationship of the pattern to the context. *Why do you think students are so concerned with even and odd rather than the total amount of decorations?*

Student H

4. When he started decorating, Cameron had 26 balloons and 19 stars. He said that he would be able to make nine pieces. Is he correct? no ✓
Explain your work.

He is wrong because the stars are even numbers and 19 is odd and the balloons are odd and 26 is even.

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Student I has a nice understanding of the unit of 5. The pattern grows in groups of 5. However it is also important to the pattern to have the right number of stars and balloons.

Student I

4. When he started decorating, Cameron had 26 balloons and 19 stars. He said that he would be able to make nine pieces. Is he correct? Yes
 Explain your work.

You do 26 balloons + 19 stars it will be 45 and every time Cameron adds a pattern He will use five shapes.

8

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Page 2

Birthday Decoration

After that you do $9 \times 5 = 45$. so the answer is Yes.

Student J does not know how to interpret the chart. He tries to add all the numbers together to get a huge number of decorations. He does not see that the elements in later steps contain the elements from the previous steps.

Student J

number of pieces	1	2	3	4	5
number of stars	2	4	6	8	10
number of balloons	3	6	9	12	15

2. Fill in the numbers in the table for three pieces.
 3. How many stars and balloons will he need to make five pieces? Write your answer in the table above.
 4. When he started decorating, Cameron had 26 balloons and 19 stars. He said that he would be able to make nine pieces. Is he correct? no
 Explain your work.

I added up all the amounts of the shapes, and I got a higher amount than 45.

8

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Page 2

Birthday Decoration

Student K is able to do the mathematics of extending the table, but has no idea how to interpret the information. The student is just practicing skip counting. In part 4 the student tries to examine the differences rather than think about the number of total pieces needed to make the design.

Student K

Cameron needs to know how many stars and balloons he will be using. He makes this table to help.

number of pieces	1	2	3	4	5	6	7	8	9
number of stars	2	4	6	8	10	12	14	16	18
number of balloons	3	6	9	12	15	18	21	24	27

- Fill in the numbers in the table for three pieces.
- How many stars and balloons will he need to make five pieces?
Write your answer in the table above.

- When he started decorating, Cameron had 26 balloons and 19 stars. He said that he would be able to make nine pieces. Is he correct?
Explain your work.

NO. 26
-19

10
 Because she subtracted 26-18 to get 9 but
 she was supposed to subtract 26-19 to get 10 so its
 no. 8

Student Task	Find and extend a geometric pattern in drawings, tables, and numbers. Be able to justify why something does or does not fit the pattern.
Core Idea 3 Patterns, Functions and Algebra	Understand patterns and use mathematical models to represent and to understand qualitative and quantitative relationships. <ul style="list-style-type: none"> • Describe and extend geometric and numeric patterns. • Represent and analyze patterns using words and/or tables • Model problem situations with objects and use representations such as graphs and tables to draw conclusions.

The mathematics of this task:

- Recognizing a pattern with different elements and extending the pattern with a drawing
- Extending the pattern using a table
- Reasoning about the number of individual decorations needed to replicate the pattern 9 times and making a justification about whether there are enough pieces
- Interpreting computations in context
- Interpreting remainders

Based on teacher observations, this is what third graders know and are able to do:

- Extend a pattern by drawing
- Skip count
- Multiply
- Use a table

Areas of difficulty for third graders:

- Understanding that 9 pieces meant 9 parts of the pattern, not separate decoration elements
- Understanding how to interpret calculations, e.g. counting by 3's there is no 26 does not explain that 1 more balloon is need to make 9 sets of 3
- Understanding that having enough decorations is not about odd or even, but on the total amount
- Understanding that its ok to have extras

Strategies used by successful students:

- Extending the table
- Using multiplication to find the number of stars and balloons needed
- Making a comparison of amount needed to amount on hand
- Drawing and counting
- Quantifying how each part of the pattern is growing

MARS Test Task 2 Frequency Distribution and Bar Graph, Grade 3

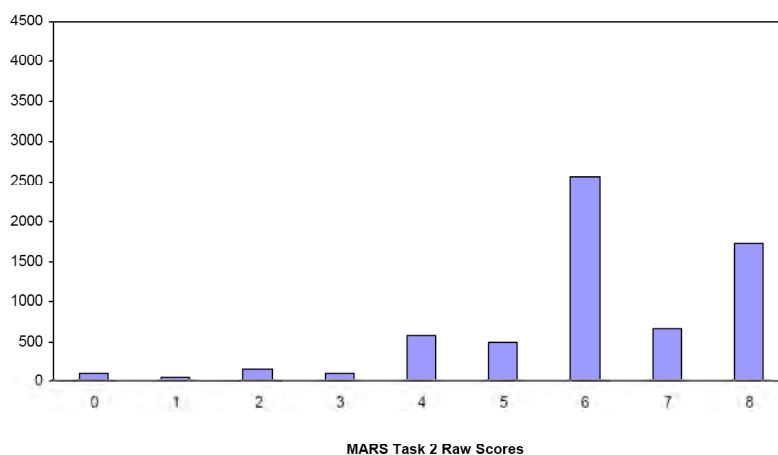
Task 2 - Birthday Decoration

Mean: 6.11 StdDev: 1.70

Table 16: Frequency Distribution of MARS Test Task 2, Grade 3

Task 2 Scores	Student Count	% at or below	% at or above
0	100	1.6%	100.0%
1	51	2.3%	98.4%
2	151	4.7%	97.7%
3	102	6.3%	95.3%
4	575	15.2%	93.7%
5	495	22.9%	84.8%
6	2557	62.7%	77.1%
7	666	73.0%	37.3%
8	1733	100.0%	27.0%

Figure 25: Bar Graph of MARS Test Task 2 Raw Scores, Grade 3



The maximum score available for this task is 8 points.

The minimum score for a level 3 response, meeting standards, is 5 points.

Most students, 83%, could extend the pattern to 3 and 5 pieces using the table. 77% could also extend the pattern by making a drawing. Some students, 27%, could also explain why there weren't enough decorations to make nine pieces. Almost 2% of the students scored no points on the task. No students in the sample scored zero.

Birthday Decorations

Points	Understandings	Misunderstandings
0	No students in the sample had this score.	<i>No information on scores lower than 4 in the sample set.</i>
4	Students could use the table to extend the pattern to 3 and 5 pieces.	5% of the students only drew the stars and left out the balloons. 4% of the students did not make a drawing.
6	Students could draw the pattern for 3 pieces and extend the pattern using the table.	Students did not know how to make a justification for part 4. Many students put answers about even and odd, not interpreting the pattern to the context. Others thought about the total number of decorations, 45, but ignored the fact that they weren't the right shapes. Many did calculations that could have led to the correct answer, but could not interpret the meaning.
8	Students could recognize a pattern and extend it using a drawing and a table. Students could make a convincing mathematical argument about why there weren't enough decorations to make 9 pieces.	

Implications for Instruction

Students at this grade level need to have opportunities to draw and extend patterns. Students should start to think about multiplication of equal size groups when working with patterns. Students need more experience working with problems in context, so they have to puzzle about the meaning of their calculations. Students need to think about comparisons, what does it mean to have enough? Too much? What is possible to fit this situation? Students need to start developing the ability to make convincing arguments. They should know to quantify their answers.

Ideas for Action Research

Students need opportunities to work with a variety of problems that require giving justifications. You might have students work a 4th grade 2003 Task, Flower Arranging, which requires logical thinking and explanations. This is a good task for thinking about constraints and how ideas match constraints. 2006 3rd grade task, Houses in a Row, is a opportunity to work with growing patterns and think about making justifications of why something is or is not possible. 2005 3rd grade task, Number Cards, gives students a chance to work with place value and making justifications. In working these tasks, think about showing different explanations and having students evaluate which explanations are more convincing. See if students can start to develop their own internal standards for making good explanations.