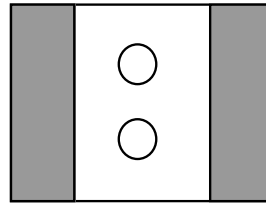


Joyce's Rugs

This problem gives you the chance to:

- describe and extend patterns

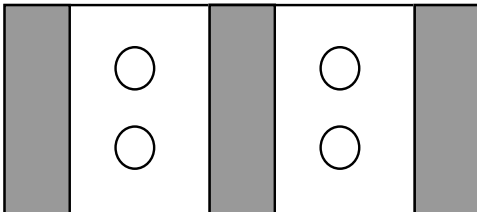
Joyce makes patterned rugs.
This is one of her designs.



Rug #1

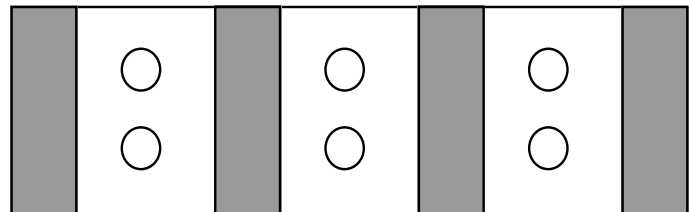
The design has 2 stripes and 2 circles.

She makes the rugs in different lengths.



Rug #2.

It has 3 stripes and 4 circles.



Rug #3.

It has 4 stripes and 6 circles.

Draw a sketch of Rug #4.

2. Here is a table that helps Joyce to work out her rug length designs.

Rug #	1	2	3	4	5
Number of stripes	2	3	4		
Number of circles	2	4	6		

Complete the table to show how many stripes and circles there are on Rug #4 and Rug #5.

3. How many circles are there on Rug #11? _____

Explain how you figured it out.

4. A rug has 24 stripes. How many circles does it have? _____

Tell how you figured it out.

Task 2: Joyce's Rugs		Rubric	
The core elements of performance required by this task are: • describe and extend patterns Based on these, credit for specific aspects of performance should be assigned as follows		points	section points
1.	Draws correct diagram for the Rug #4.	1	1
2.	Gives correct answers for Rugs # 4 and #5: 5 6 8 10	1 1	2
3.	Gives correct answer: 22 Gives a correct explanation or draws diagrams to show that each rug has twice as many circles as its rug number.	1 1	2
4.	Gives correct answer: 46 Explanations such as: The pattern number will be 23 because there is one more stripe than the pattern number. Each pattern number has twice that number of circles. So $23 \times 2 = 46$.	1 1	3
Total Points			8

8

Joyce's Rugs

Work the task. Look at the rubric. What are the major mathematical ideas in this task?

What strategies do you think successful students might use to solve for the number of circles in part 4?

Look at student work for part 4. How many of your students put:

46	42	44	48	25	More than 50	Other

What might the student have been thinking to get an answer of 42? 44? 48? 25? What is the mathematical idea each student is confused about? How is each error an indicator of a different mathematical misconception?

Now look at student explanations for part 4. How many of your students:

Used drawing and counting: _____ Did any of these students:

- Add too many stripes?
- Make an error because their picture was in two rows? How did continuing the drawing on the second row contribute to the error? What did the student need to consider in order to use this strategy accurately?

Continued the table or used an adding-on strategy: _____

Explained their calculations and worked backwards: _____

Wrote a verbal rule that is generalizable for any number: _____

How often do students get opportunities to make generalizations? How does this help them at later grades?

Looking at Student Work on Joyce's Rugs

Student A is able to write rules for parts 3 and 4 that are generalizable to any number of rugs. These rules could easily be turned into algebraic expressions. *If r = rug number, s = number of stripes, and c = number of circles, could you change Student A's rules into algebraic expressions?*

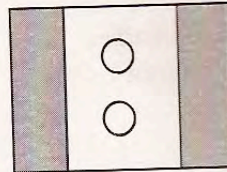
Student A

Joyce's Rugs

This problem gives you the chance to:

- describe and extend patterns

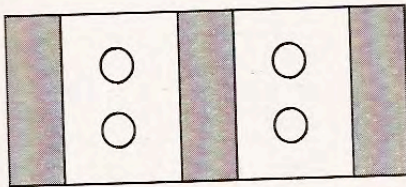
Joyce makes patterned rugs.
This is one of her designs.



Rug #1

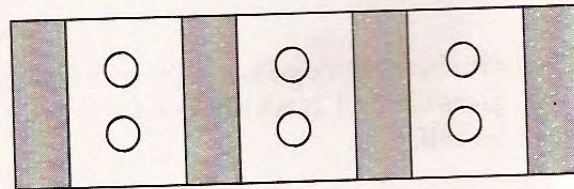
The design has 2 stripes and 2 circles.

She makes the rugs in different lengths.



Rug #2.

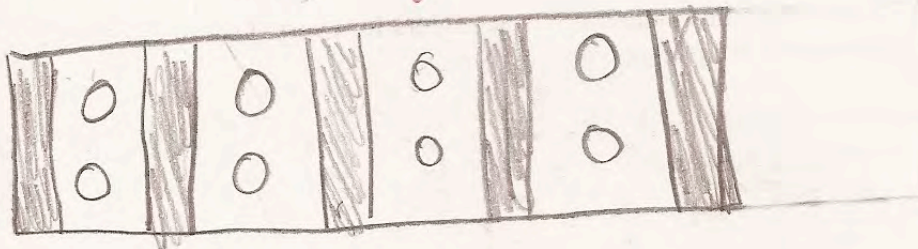
It has 3 stripes and 4 circles.



Rug #3.

It has 4 stripes and 6 circles.

Draw a sketch of Rug #4.



2. Here is a table that helps Joyce to work out her rug length designs.

Rug #	1	2	3	4	5
Number of stripes	2	3	4	5	6
Number of circles	2	4	6	8	10

Complete the table to show how many stripes and circles there are on Rug #4 and Rug #5.

Student A, part 2

3. How many circles are there on Rug #11?

22 circles ✓ 7

Explain how you figured it out.

Because the pattern shows that double the rug # = its circles: $11 \times 2 = 22$
(double) (circles)

4. A rug has 24 stripes. How many circles does it have?

46 circles ✓

Tell how you figured it out.

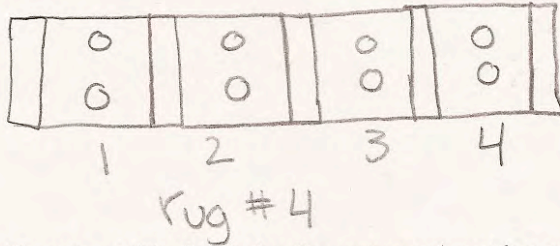
The pattern shows that there is one more stripe than the rug # so the rug with 24 stripes must be rug #23. Double of 23 = 46 circles.

$$\begin{array}{r} 23 \checkmark \\ + 23 \\ \hline 46 \end{array}$$

Student B explains the rules in part 3 and 4 and uses the table in part 2 to test the conjectures or show where the rules come from, relating the numerical representations to the physical representations.

Student B

Draw a sketch of Rug #4.



2. Here is a table that helps Joyce to work out her rug length designs.

Rug #	1	2	3	4	5
Number of stripes	2	3	4	5	6
Number of circles	2	4	6	8	10

Complete the table to show how many stripes and circles there are on Rug #4 and Rug #5.

$4 + 1 = 5$
 $4 \times 2 = 8$
 $5 + 1 = 6$
 $5 \times 2 = 10$

3. How many circles are there on Rug #11?

22 o's

Explain how you figured it out.

If there are 20 per rug and 11 rugs were put together than you would have to multiply 2×11 witch = 22 o's.

4. A rug has 24 stripes. How many circles does it have?

46 o's

Tell how you figured it out.

Every rug has 1 more stripe than its # how many rugs are put together. Because every rug has 2x as many o's I had to multiply 23 by 2 witch got me to an answer of 46 o's.

$$\begin{array}{r} 23 \\ \times 2 \\ \hline 46 \end{array}$$

Student C has a different strategy for extending the pattern in part 3. Why does this method work? Could you use this strategy to find the number of circles for rug #20? Student C is able to calculate the correct number of circles in part 4 and shows a good understanding of the situation, but struggles with the need for precise language to describe the pattern. How could the student revise the rule to match his calculations?

Student C

3. How many circles are there on Rug #11? 22 ✓

Explain how you figured it out.

2 circles are added each time. $11 - 5 = 6$

$6 \times 2 = 12$ ~~10~~ $+ 2 = 10$ $+ 2 = 12$ $+ 2 = 14$ $+ 2 = 16$ $+ 2 = 18$ $+ 2 = 20$ $+ 2 = 22$ ✓

4. A rug has 24 stripes. How many circles does it have? 46 ✓

Tell how you figured it out.

a rug has twice as many circles as stripes
plus one stripe ✓

~~X~~ ~~12~~ ~~12~~

Student D simplifies the multiplication in part 3 to use friendly numbers. *What is the multiplication being shown in this student's strategy?* In part 4 the student has simplified the rug into key elements and eliminated unnecessary information. However, because the pattern is so lengthy the student seems to have forgotten part of the pattern? *What is the error in the drawing? Do you think the student may have made the drawing to avoid multiplication? Why or why not?*

Student D

3. How many circles are there on Rug #11? 22 ✓ |

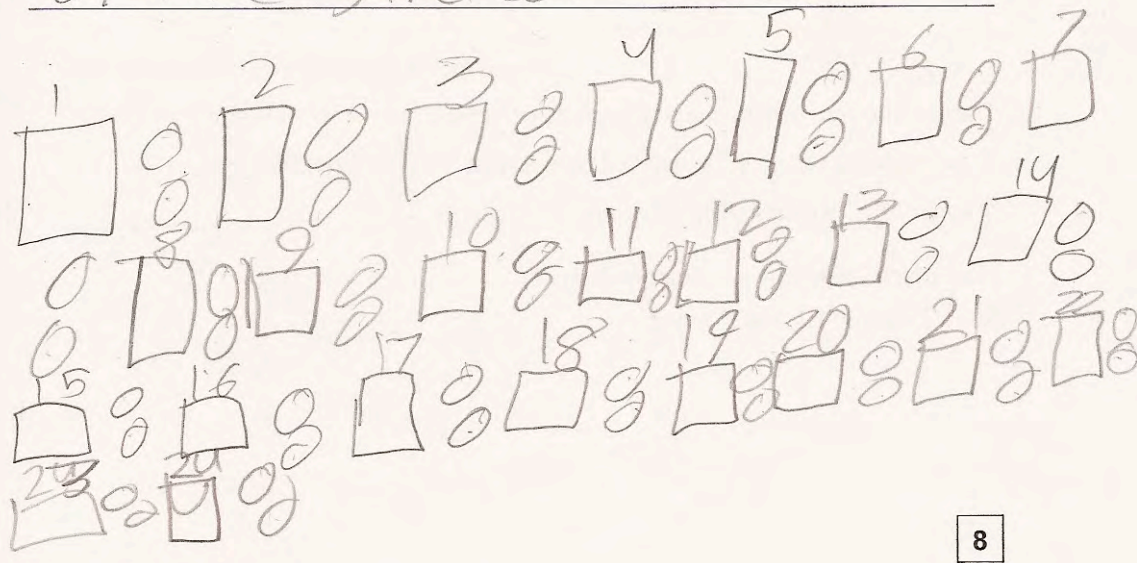
Explain how you figured it out.

10 circles is rug number #5
so if you times 10 by 2 you get ✓ |
20 and you add 2.

4. A rug has 24 stripes. How many circles does it have? 48 x 0 ○

Tell how you figured it out.

I did the rug design with x 0 ○
24 stripes then counted all ✓ |
of the circles.



Student E uses a table to extend the pattern in part 3. While the student “sees” a pattern, the pattern is never described. There isn’t evidence enough to understand if the student is just looking at an “add-on” to the numbers in the table or thinking about stripes, circles, and rug numbers. In part 4 the student makes in error an trying to continue the pattern on a second line. *What has the student done incorrectly in the drawing? Why does this lead to an incorrect answer?*

Student E

it has 4 stripes and 6 circles.

Draw a sketch of Rug #4.

2. Here is a table that helps Joyce to work out her rug length designs.

Rug #	1	2	3	4	5	6	7	8	9	10
Number of stripes	2	3	4	5	6	8	10	12	14	16
Number of circles	2	4	6	8	10	12	14	16	18	20

Complete the table to show how many stripes and circles there are on Rug #4 and Rug #5.

Student E, part 2

3. How many circles are there on Rug #11?

22 ✓. ()

Explain how you figured it out.

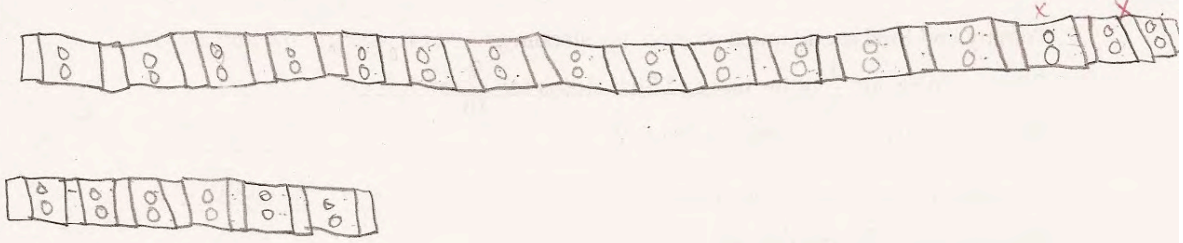
I make a patterns
and make it into 11
and it give me 22.

4. A rug has 24 stripes. How many circles does it have?

44 x. 0

Tell how you figured it out.

I make a patterns ✓
and I had 24 stripes and
44 circles and that is my corte
44.



Look at work for Student F. Using a chart is a good strategy, but it may distract students from thinking about the relationships being described by the numbers. *What relationship has the student forgotten about when attempting to do part 4 of the task? What part of the table is the student thinking about when doing the 24×2 ?*

Student F

3. How many circles are there on Rug #11?

Explain how you figured it out.

Rug	1	2	3	4	5	6	7	8	9	10	11
Stripe	2	3	4	5	6	7	8	9	10	11	12
Circle	2	4	6	8	10	12	14	16	18	20	22

22

22. I made a chart. Also the circle are always two more than before so you have to multiply by 2.

4. A rug has 24 stripes. How many circles does it have?

Tell how you figured it out.

$24 \times 2 = 48$

48

Need to multiply it by 2 so
 24×2 that will equal 48

Student G is seeing the pattern of doubling, but thinks about doubling using addition rather than multiplication. *How do we help transition students into thinking about equal groups as multiplication? Why is this important?* While the student knows the stripes are more than the rug number, the student can't use that information to work backwards and so chooses the incorrect operation. Students at this grade level need to not only understand how to do computations, but also have an understanding of the meaning of operations with whole numbers. An important part of algebraic thinking is the idea of doing and undoing. *What kinds of activities can help students to see the relationships between addition and subtraction or multiplication and division?*

Student G

3. How many circles are there on Rug #11?

22 circles ✓

Explain how you figured it out.

The number of circles is adding ~~itself~~ ✓ it self. 0!

Example: Rug ~~#4~~, $4+4=8$ circles.

4. A rug has 24 stripes. How many circles does it have?

50 circles ✗

Tell how you figured it out.

The stripes is 1 more than the number rug 0.
and then the Rug ~~#2~~ has to be ~~25~~ ✓ 25.
so you add $25+25=50$ 0

Student H may also be struggling with multiplication. The student is able to use multiplication for 11×2 , but then seems to want to calculate 24×2 but uses skip counting instead. What might be your next steps with this student?

Student H

3. How many circles are there on Rug #11?

22 circles ✓ 1 1

Explain how you figured it out.

If you do $2 \times 11 = 22$ and it was the 11's one. ✓ 1 1

$$\begin{array}{r} 11 \\ \times 2 \\ \hline 22 \end{array}$$

4. A rug has 24 stripes. How many circles does it have?

50 X 0 0

Tell how you figured it out.

You count down to 24 to 0 and circles go 0 0

by 2. So count down ✓ 1 1

Handwritten skip counting sequence:

24 22
23
22 19 18 17 16 15 14 13 12 11 10 9 8
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34
26 28 30 32 34 36 38 40 42 44 46 48 50

Fifth Grade

5th Grade

Task 2

Joyce's Rugs

Student Task	Draw and extend a pattern. Justify how to extend the pattern. Use inverse operations to solve a problem.
Core Idea 3 Patterns, Functions and Algebra	Understand patterns and use mathematical models such as algebraic symbols and graphs to represent and understand quantitative relationships.

Based on teacher observation, this is what fifth graders knew and were able to do:

- Extend the pattern by drawing
- Extend the pattern using a table
- Find the number of circles given the rug number

Areas of difficulty for fifth graders:

- Working backwards from the number of stripes to find the number of circles
- Writing generalizable rules
- Some students struggled with multiplicative thinking.

Strategies used by successful students:

- Thinking about how the circles, stripes, and rug numbers are related seemed to help students write rules for any number of circles and for working backwards.
- Using a table seemed to distract students from thinking about the relationships.
- Relying on drawing, while a strategy that could work, often resulted in errors because of the size of the pattern. Students with more efficient strategies were generally more successful.

MARS Test Task 2 Frequency Distribution and Bar Graph, Grade 5

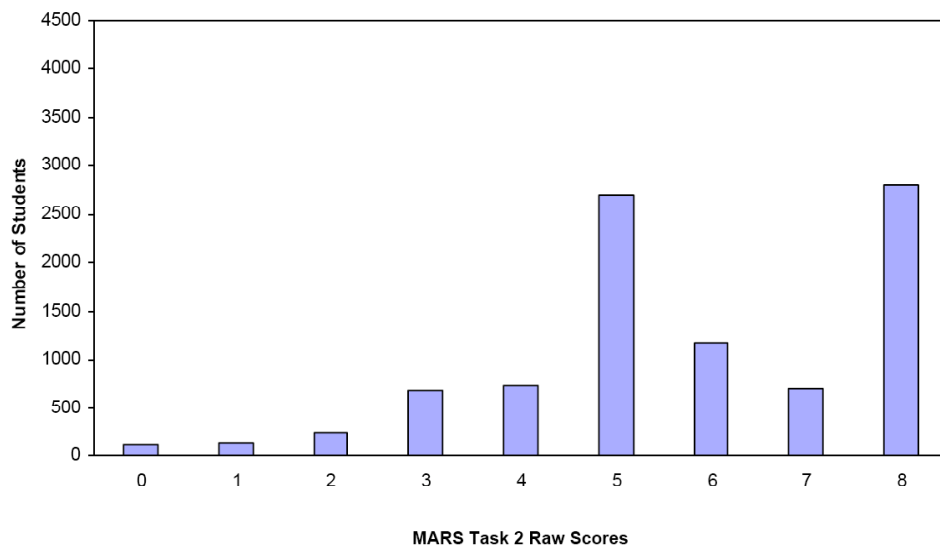
Task 2 - Joyce's Rugs

Mean: 5.76 StdDev: 1.93

Table 26: Frequency Distribution of MARS Test Task 2, Grade 5

Task 2 Scores	Student Count	% at or below	% at or above
0	125	1.3%	100.0%
1	128	2.7%	98.7%
2	231	5.2%	97.3%
3	679	12.5%	94.8%
4	727	20.4%	87.5%
5	2703	49.6%	79.6%
6	1177	62.3%	50.4%
7	699	69.8%	37.7%
8	2798	100.0%	30.2%

Figure 35: Bar Graph of MARS Test Task 2 Raw Scores, Grade 5



The maximum score available for this task is 8 points.

The minimum score for a level 3 response, meeting standards, is 5 points.

Most students, 95%, could draw the rug for the 4th pattern and use a table to extend the pattern. Many students, 80%, could extend the pattern by drawing and using a table, and find the number of circles for rug#11. 30% of the students could meet all the demands of the task, including working backwards from the number of stripes to the number of circles. This involved understanding that the number of stripes was different from the rug number and making an appropriate adjustment to the given number before finding the circles. 1% of the students scored no points on this task.

Joyce's Rugs

Points	Understandings	Misunderstandings
0	No students in the sample scored 0 or 1.	Some students forgot the final stripe. Some students made 3 circles in every white patch. Some students put 2 stripes in the middle of the rug. Some students drew rug 5 instead of rug 4.
3	Students could extend the pattern in a drawing and use a table.	Some students had difficulty finding the number of circles for rug #11. 3% thought the answer was 12, thinking of the extra stripe instead of the circles. Other common answers: 24 and 33.
5	Students could extend the pattern in a variety of ways from 3 to 4 to 5 to 11.	Students struggled with the idea of working backwards. 17% just doubled the number of stripes for an answer of 48 circles. 9% thought the answer was 44. 3% added one more section instead of subtracting so they had an answer of 50. Some students had answers of 11, 12, 25, and 520.
8	Students could extend a geometric pattern and work backward from a number of elements in the pattern to find the quantity of another set of elements in the pattern. Many successful students were able to talk about the relationships between pattern number and the stripes and pattern number and the number of circles.	Students using drawing and counting had more difficulty working backwards. The same is true for students trying to use a table, who often saw the doubling in the table but didn't clearly identify which quantity was doubling.

Implications for Instruction

Students need opportunities to draw patterns and talk about the geometric elements of the design. The things students notice in the process of drawing helps them to identify the relationships in the pattern. It also helps them to notice the structural elements that help to move them through the van Hiele levels. While most students were able to successfully draw the 4th rug, their papers show evidence of many erasures. Students would make a large bar and then not be able to estimate the size of the pieces to make it match the number of parts needed for rug four. This development of visual estimation and portioning is critical to their later development of understanding area models for fractions and reason about geometric shapes in general. Students need opportunities to make diagrams.

Connected with making models is talk about the relationships they see. What are the elements that make up the pattern? How is the number of stripes connected or related to the rug number? How is the number of circles related to the rug number? Looking for these connections help students to build rules that will translate into formulas at later grade levels. Students who relied on strategies for drawing and counting often made errors because they duplicated part of the pattern when the drawing took up two or more rows or they got so busy drawing they forgot part or added extra parts to the pattern. Students should be encouraged to move to more sophisticated strategies.

Students at this grade level should be transitioning from additive thinking to multiplicative thinking. They should start to see that if something is growing by an equal-size unit each time that multiplication is involved. They should also have a firm grasp of the meaning of operations with whole numbers, so that they can start to grapple with how operations affect rational numbers. They should be able to think about doing and undoing in terms of choosing operations.

Ideas for Action Research

One useful strategy when student work does meet your expectations is to use sample work to promote deeper thinking about the mathematical issues in the task. In planning for re-engagement it is important to think about what is the story of the task, what are the common errors and what are the mathematical ideas I want students to think about more deeply. Then look through student work to pick key pieces of student work to use to pose questions for class discussion. Often students will need to have time to rework part of the task or engage in a pair/share discussion before they are ready to discuss the issue with the whole class. This reworking of the mathematics with a new eye or new perspective is the key to this strategy.

In this task, there are two issues that might be interesting to explore or re-engage in. The first is the variety of strategies used in part 3. Can we use some of these strategies to help move students away from drawing and counting or additive thinking to finding rules that use multiplication?

Try posing a question like this to the class.

Martha solves part 3 by: $6 \times 2 = 12$ and then $12 + 10 = 22$. What do you think Martha is doing? Why does she multiply by 2? Where do you think the 10 comes from? Will her strategy work with other numbers? Could you use her strategy to find the number of circles needed for rug #20?

After some class discussion, if a general rule does not come up pose this additional question.

Roger thinks there is a simpler way to find the number of circles for any rug number. Can you help Roger find a rule?

How do these questions push students to rethink or re-engage with the mathematics of the task? How do the questions foster discussion about the big ideas of using variables? How does the struggle to justify and convince help students to clarify their own thinking or look at the mathematics from a different perspective? Does this prompt challenge the student who got it right the first time?

Another interesting issue is the working backwards in part 4. Consider posing a question like:

Anna is excited by the idea of using rules. She wants to see if there is a rule for finding the number of circles if you know the number of stripes. Can you use Anna's table to help her find a rule?

2. Here is a table that helps Joyce to work out her rug length designs.

Rug #	1	2	3	4	5
Number of stripes	2	3	4	5	6
Number of circles	2	4	6	8	10

Complete the table to show how many stripes and circles there are on Rug #4 and Rug #5.

$4 + 1 = 5$
 $4 \times 2 = 8$
 $5 + 1 = 6$
 $5 \times 2 = 10$

Can you use this rule to find the number of circles if there are 50 stripes?

Is there some piece of work from your class that you could make into a good question for class discussion? What mathematics are you hoping students will discover from your question?