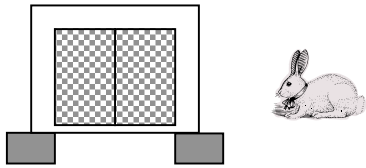

Roger's Rabbits

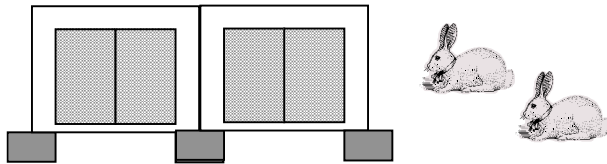
This problem gives you the chance to:

- identify and extend patterns
 - work with tables
-

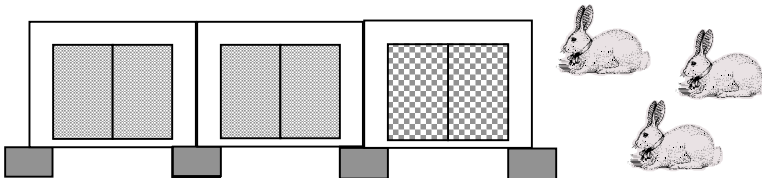
Roger keeps pet rabbits. He keeps them in a row of rabbit hutches. The hutches are on blocks so that they don't get damp.



This is hutch #1.
It is for one rabbit.
It has 2 doors and 2 blocks.



This is hutch #2.
It is for two rabbits.
It has 4 doors and 3 blocks.



This is hutch #3.
It is for three rabbits.
It has 6 doors and 4 blocks.

1. Describe hutch #4.

2. Fill in the empty spaces in the table below.

Hutch #	1	2	3	4
Number of doors	2	4		
Number of blocks	2	3		

3. How many doors will be needed for hutch # 8? _____
Explain how you figured this out.

How many blocks will be needed for hutch #8? _____

4. Roger says that for hutch # 12 he will need 11 blocks. Roger is wrong.
How many blocks will he need? _____

Explain how you figured this out.

Roger's Rabbits	Rubric																
<p>The core elements of performance required by this task are:</p> <ul style="list-style-type: none"> • identify and extend patterns • work with tables <p>Based on these, credit for specific aspects of performance should be assigned as follows</p>	points	section points															
1. Gives a correct description: 4 rabbits, 8 doors and 5 blocks.	2 x 1	2															
<p>2. Completes the table correctly.</p> <table border="1" data-bbox="261 548 1206 680"> <tr> <td>Hutch #</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>Number of doors</td> <td>2</td> <td>4</td> <td>6</td> <td>8</td> </tr> <tr> <td>Number of blocks</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> </tr> </table> <p><i>Partial credit</i> 2 correct numbers</p>	Hutch #	1	2	3	4	Number of doors	2	4	6	8	Number of blocks	2	3	4	5	2 (1)	2
Hutch #	1	2	3	4													
Number of doors	2	4	6	8													
Number of blocks	2	3	4	5													
<p>3. Gives correct answer: 16 doors</p> <p>Gives correct explanation such as: the number of doors is twice the hutch number or draws diagram.</p> <p>Gives correct answer: 9</p>	1 1 1	3															
<p>4. Gives correct answer 13 blocks.</p> <p>Gives correct explanation such as: He will need two blocks for hutch number 1 and then one block for each of the next blocks.</p>	1 1	2															
Total Points		9															

Roger's Rabbits

Work the task. Look at the rubric? What are the big mathematical ideas in this task? What strategies do you anticipate your students might use to solve this task?

Were your students able to give a description of the hutch #4?
Did their descriptions help them to identify the patterns in the task?

Look at their work for part 3. Were students able to identify the number of doors for hutch # 8 as 16? _____ 8? _____ other? _____

- How many students drew a picture?
- Continued the table or did repeated addition by 2's?
- How many gave a verbal rule, such as the number of doors is t times the pattern #?
- How many students multiplied by 2?

Now look at their work for the number of blocks. How many students put:

9	8	10	12	16	Other

How do you think the students were thinking about the blocks who made some of these errors? What attributes of the pattern were they not seeing? Where were they making faulty generalizations?

Now look at the work for part 4. While many students could find the number of blocks in part 3, they often did not use that same strategy for part 4. When looking at errors check to see if they were using the same or a different strategy. How many of your students put:

9	8	12	15	16	24	13	20	other

Why do you think finding the blocks was more difficult here than in part 3?

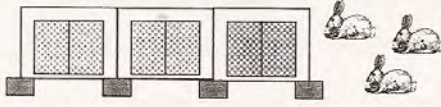
In your classroom, how do you help students move from drawing and counting strategies to thinking about describing how the pattern grows and using multiplication?

What is the benefit for the student? How does the mathematics of finding a rule contribute to the mathematics needed in later grade levels?

Looking at Student Work on Roger's Rabbits

Student A searches for patterns and relationships in the numbers. Notice how the table is labeled to show how the patterns grow in part 2. Then above part 3 the student verifies the relationship between the hutch number and the number of doors and then writes a written rule to describe the relationships. Notice the habits of mind to test conjectures, label work and organize work, and look for relationships in the numbers.

Student A



This is hutch #3.
It is for three rabbits.
It has 6 doors and 4 blocks.

1. Describe hutch #4.
Hutch #4 would have 4 rabbits, 8 doors, and 5 blocks.

2. Fill in the empty spaces in the table below.

Hutch #	1	2	3	4
Number of doors	2	4	6	8
Number of blocks	2	3	4	5

counting by 2's
counting by 2's
counting by 1's but starts on 2

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Hutch #1 + #1 = 2 doors. 1, 2, 3, 1, 1, 2
Hutch #2 + #2 = 4 doors. 1, 2, 3, 2, 2, 4
Hutch #3 + #3 = 6 doors. 2, 4, 6, 3, 3, 6

3. How many doors will be needed for hutch # 8? 16 doors
Explain how you figured this out.
Because when I looked at the others the hutch # doubled equals the # of doors.

How many blocks will be needed for hutch #8? 9 blocks
Hutch #1 has 2 blocks.
Hutch #2 has 3 blocks.
Hutch #3 has 4 blocks.

4. Roger says that for hutch # 12 he will need 11 blocks. Roger is wrong.
How many blocks will he need? 13 blocks
Explain how you figured this out.
Because each block is 1 number greater than the hutch number, like this:

Hutch #1	2 blocks	2 doors
Hutch #2	3 blocks	4 doors
Hutch #3	4 blocks	6 doors
Hutch #4	5 blocks	8 doors

Student B is able to answer all the questions in the task, but the student is relying on strategies from earlier grade levels. The student needs to draw and count every part of the pattern or use repeated addition. Notice that the student doesn't think to continue a drawing or continue a table, but starts from scratch each time. In the work by question one the student seems to understand the idea of multiplying by 2 doors for each hutch, but the student doesn't seem to know how to make the next developmental step to using the rule to replace tables and drawings. *How do we help students leave behind comfortable strategies and make the effort to learn new ones?*

Student B

1. Describe hutch #4.
 Hutch #4 would have 8 doors and 5 blocks and four rabbits.

2. Fill in the empty spaces in the table below.

Hutch #	1	2	3	4
Number of doors	2	4	6	8
Number of blocks	2	3	4	5

How many doors will be needed for hutch # 8? 16 doors
 Explain how you figured this out.
 I did it by looking at the chart below and noticed the amt. of doors kept counting by 2's and it went 2, 4, 6, 8, 10, 12, 14, 16.
 #1 #2 #3 #4 #5 #6 #7 #8

How many blocks will be needed for hutch #8? 9 blocks

4. Roger says that for hutch # 12 he will need 11 blocks. Roger is wrong.
 How many blocks will he need? 13 blocks
 Explain how you figured this out.
 I drew it out with 15 squares and put the blocks were they went and then I counted the blocks.

Student C is able to do parts 1, 2 correctly. The student is able to see the doubling pattern for the doors. However the student doesn't use the information in part 1 or 2 to help find the number of blocks. Student does not observe that the first hutch has two blocks. *What questions for self-talk might have helped this student? How do we help students move from noticing how a pattern grows to noticing the relationships between two variables (hutch # and # of blocks)? Do we give students enough opportunities to explain how they know their answers are true? To listen to the arguments of other students and hear other points of view? To debate different conjectures?*

Student C

3. How many doors will be needed for hutch # 8? 16 ✓ ✓

Explain how you figured this out.

Beccause each have two ✓
doors

How many blocks will be needed for hutch #8? 8 × ×

4. Roger says that for hutch # 12 he will need 11 blocks. Roger is wrong.

How many blocks will he need? 12 × ×

Explain how you figured this out.

Beccause each Box have one ×
×

Student D has to draw pictures, but doesn't have that diagram literacy to think about what are the important or essential features. The student draws the screens on the door and even draws in the rabbits. The student understands how the pattern grows, but in the table doesn't use enough detail (omits the hutch # row) to find the correct solution for the number of blocks in part 4. Notice the student uses the table successfully for the number of blocks in part 3, but doesn't notice the relationship. The student is not thinking about patterns, but just number crunching.

Student D

1. Describe hutch #4.

4 rabbits, 8^v doors, 5 blocks. ✓ ✓ 2¹



2. Fill in the empty spaces in the table below.

Hutch #	1	2	3	4
Number of doors	2	4	6 ✓	8 ✓
Number of blocks	2	3	4 ✓	5 ✓

10 12 14 16 ✓
6 7 8 9 10 11 12

3. How many doors will be needed for hutch # 8? 16 ✓

Explain how you figured this out.

I added onto the table (left) until I got to hutch #8. ✓

How many blocks will be needed for hutch #8? 9 ✓ ✓

(I added onto the table)

4. Roger says that for hutch # 12 he will need 11 blocks. Roger is wrong.

How many blocks will he need? 12 × 0 ×

Explain how you figured this out.

I again added onto your table (left). ✓

Student E is able to see the pattern for the blocks, but the student looks at the picture and sees one door instead of two doors per hutch. The student doesn't use the verbal clues by the side of the drawings.

Student E

1. Describe hutch #4.

hutch #4 has 5 blocks and 4 doors

2. Fill in the empty spaces in the table below.

Hutch #	1	2	3	4
Number of doors	2	4	3 ^x	4 ^x
Number of blocks	2	3	4 [✓]	5 [✓]

3. How many doors will be needed for hutch # 8?

8^x

Explain how you figured this out.

well its the same pattern as for the other ones

How many blocks will be needed for hutch #8?

19[✓]

4. Roger says that for hutch # 12 he will need 11 blocks. Roger is wrong.

How many blocks will he need? 13[✓]

Explain how you figured this out.

because I used the pattern for the past hutch's

Student F does not examine the physical model carefully. In part 1 the student may think the blocks are 2 less than the hutch number. The student is able to find the number of blocks in part 3, but there is no work to show where the number came from. Then in part 4 the student doubles to find the number of blocks. The thinking is not consistent. *What would be your next steps with this student?*

Student F

1. Describe hutch #4.

It is for 4 rabbits. It has 8 doors and 6 blocks.

2. Fill in the empty spaces in the table below.

Hutch #	1	2	3	4
Number of doors	2	4	6 ✓	8 ✓
Number of blocks	2	3	4 ✓	6 ✗

How many doors will be needed for hutch # 8?

16 ✓

Explain how you figured this out.

I multiplied eight times 2 and got 16.

How many blocks will be needed for hutch #8?

9 blocks ✓

Roger says that for hutch # 12 he will need 11 blocks. Roger is wrong.

How many blocks will he need?

24 blocks ✗

Explain how you figured this out.

I multiplied 2 x 12 and got 24. ✗

Like Student F, Student G does not give enough detail to understand what he is thinking. Questions 1 and 2 are correct, but what is added to get 12? The student can find the number of blocks in part 3, but uses a different rule to find the blocks in part 4.

Student F

3. How many doors will be needed for hutch # 8? 12^x

Explain how you figured this out.

I did addition

^ ^

How many blocks will be needed for hutch #8? 9 ✓

4. Roger says that for hutch # 12 he will need 11 blocks. Roger is wrong.

How many blocks will he need? ~~10~~ x

x

Explain how you figured this out.

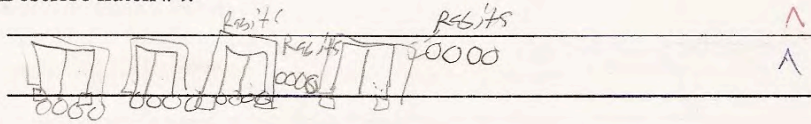
More addition x

x

Student G does not understand the basics of the pattern. The hutches are drawn disconnected, so that even a counting strategy will not help the student solve the pattern. *What type of help is available for this student at your school? Where would you go with this student in terms of providing fundamental help?*

Student G

1. Describe hutch #4.



2. Fill in the empty spaces in the table below.

Hutch #	1	2	3	4
Number of doors	2	4		
Number of blocks	2	3		

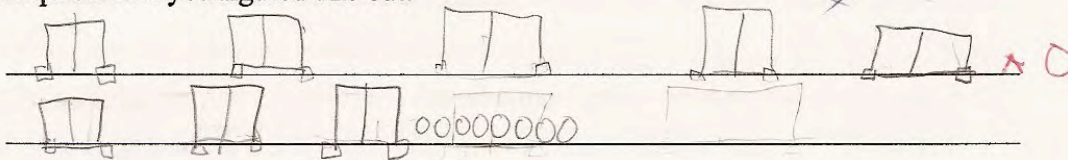
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Page 2

Fourth Grade - Roger's Rabbits

3. How many doors will be needed for hutch # 8? 8

Explain how you figured this out.

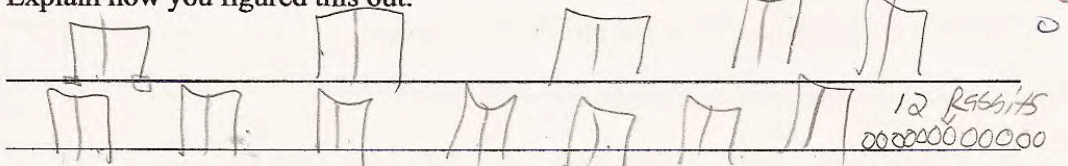


How many blocks will be needed for hutch #8? 8

4. Roger says that for hutch # 12 he will need 11 blocks. Roger is wrong.

How many blocks will he need? 12

Explain how you figured this out.



<i>Student Task</i>	Identify and extend patterns. Work with tables.
Core Idea 3 Patterns, Functions, and Algebra	<p>Understand patterns and use mathematical models to represent and to understand qualitative and quantitative relationships.</p> <ul style="list-style-type: none"> • Represent and analyze patterns and functions using words, tables, and graphs. • Find the results of a rule for a specific value. • Use inverse operations to solve multi-step problems.

The mathematics of this task:

- Extend geometric patterns and be able to describe the growth
- Use a table to continue a growth pattern
- Extend the pattern using rules, look for relationships between two variables (hutch # and number of doors & hutch # and number of blocks)
- Recognize doubling patterns
- Understand a pattern with a constant

Based on teacher observation, this is what fourth graders knew and were able to do:

- Extend the table
- Count by 2's
- Recognize that there was a pattern and verbalize it

Areas of difficulty for fourth graders:

- Over-generalizing the rule for both parts, if they doubled for doors they doubled for blocks
- Understanding that the number of blocks was not the same as the hutch #
- Consistency in finding the number of blocks (using different rules to find blocks in part 3 and 4)

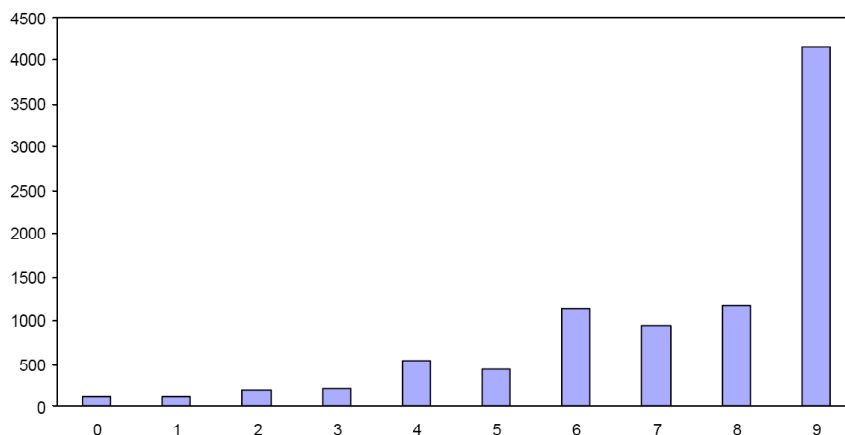
Task 2 - Roger's Rabbits

Mean: 7.26 StdDev: 2.20

Table 21: Frequency Distribution of MARS Test Task 2, Grade 4

Task 2 Scores	Student Count	% at or below	% at or above
0	124	1.4%	100.0%
1	116	2.6%	98.6%
2	202	4.9%	97.4%
3	222	7.3%	95.1%
4	549	13.4%	92.7%
5	449	18.3%	86.6%
6	1144	30.9%	81.7%
7	944	41.3%	69.1%
8	1169	54.2%	58.7%
9	4154	100.0%	45.8%

Figure 30: Bar Graph of MARS Test Task 2 Raw Scores, Grade 4



The maximum score available for this task is 9 points.

The minimum score for a level 3 response, meeting standards, is 5 points.

Most students, 93%, could explain the number of doors and blocks in hutch 4 and extend the pattern using a table. Many students 87% could also find the number of doors for hutch #8. 81% could explain how they found the number of doors. Almost half the students, 46%, could meet all the demands of the task including seeing that the number of blocks was always one more than the hutch number. Less than 2% of the students scored no points on the task. All the students in the sample with this score attempted the task.

Roger's Rabbits

Points	Understandings	Misunderstandings
0	All the students in the sample with this score attempted the task.	Some students described hutch 3 in part 1. Some students thought their were 6 blocks for hutch #4.
4	Students could describe the number of doors and blocks needed for hutch #4 and could use the table to extend the pattern.	Some students did not attempt to do the table. Some thought both patterns were growing by 2's. Some students thought both patterns were growing by 1.
5	Students could extend the pattern using a table and using a description. They could also give the number of doors needed for hutch #8.	They didn't know how to explain their thinking for finding the number of doors.
6	Students could extend the pattern using a table and using a description. They could also give the number of doors needed for hutch #8 and explain their thinking.	Students could not find the number of blocks for hutch #8. 7% of the students thought there would be 10 blocks. 5% thought there would be 8 blocks. 3% thought there would be 12 blocks.
7	Students could extend the pattern using words and a table. They understood that the doors doubled. Students could find the number of blocks in part 3, but not in part 4.	8% of the students thought the number of blocks for hutch #12 was 12. 5% thought there would be 24 blocks. 3% thought there would be 9 blocks.
9	Students could extend a pattern in words, tables, and using rules. Good students looked for connections between variables.	Struggling students still relied on drawing and counting or adding on to extend the pattern. They weren't asking themselves questions to dig deeper into the patterns to find the relationships, like the number of doors is double the hutch number or the number of blocks is one more than the hutch number.

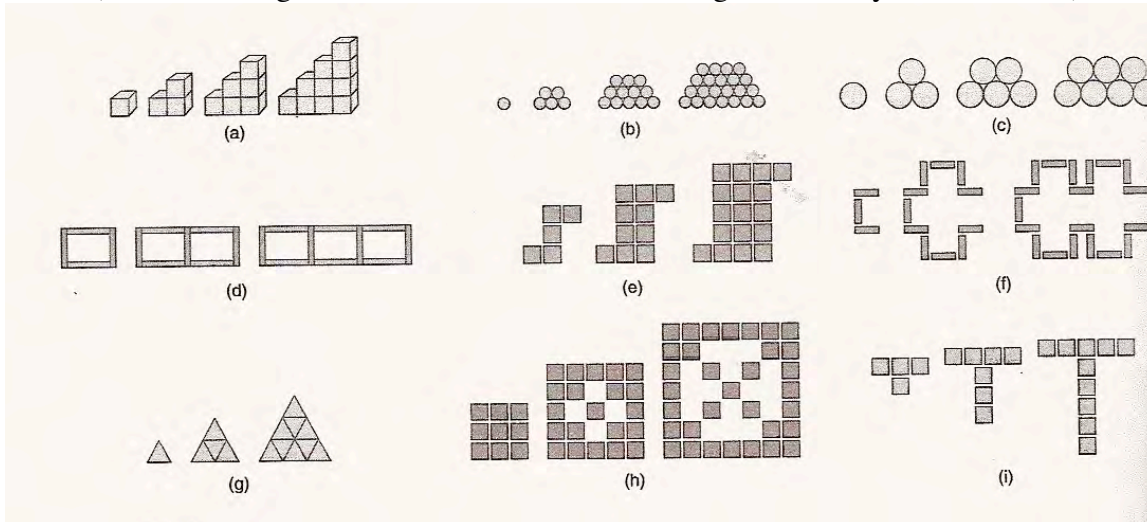
Implications for Instruction

Students need to look at visual patterns and be able to describe how they are growing. At second grade and third grade students have been able to draw and extend patterns by counting and repeated addition. At this grade level students should start to think more deeply about the patterns. Students should develop self-talk, with questions such as, “what is growing?” and “what is staying the same?” They should be starting to examine relationships between parts of the pattern. If I know the hutch number or pattern number, how can I find the number of doors or the number of blocks? If I know the number of blocks how can I find the hutch number? Students should start to make generalizations with words to find for any part of the patterns without drawing and counting or using repeated addition. Part of pattern recognition is to look for rules.

Students should be able to fill in a simple table about the number of doors and blocks needed for different numbers of rabbit hutches. Students should be comfortable recognizing patterns that grow by 2's. Students should be thinking about things that grow at a constant amount, like going up by 2's, as a multiplication pattern.

Ideas for Action Research – Searching for Deeper Patterns, Learning Self-talk

Students need to learn a process for thinking about patterns in deeper ways. Consider the patterns below (from Teaching Student-Centered Mathematics, grades 3-5 by Van de Walle):



Students should be given materials and paper to build and draw what comes next in the pattern. They should start to develop questions about how the pattern grows. What is new and different about the next shape? What changes from one piece in the pattern to the next? Ask students to explain why their extension follows the pattern. Students should also look for rules. For example, students might not be able to find a mathematical rule to extend pattern b, but they should be able to give a verbal rule for drawing the 10th term without needing to build every one in between. Students at this grade level should also start to justify their rules by giving trying their rules on the numbers from a table to see if it follows the pattern.