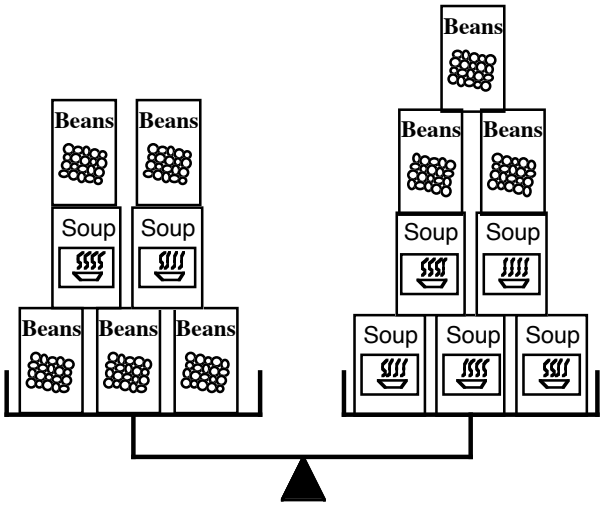
Soup and Beans

This problem gives you the chance to:

- make an equation and solve a problem

The weight of one can of beans is x ounces.

The weight of one can of soup is y ounces.



1. Write an expression for the weight of the cans on the left hand side of the weighing scales.

   _____

2. Write an expression for the weight of the cans on the right hand side of the weighing scales.

   _____

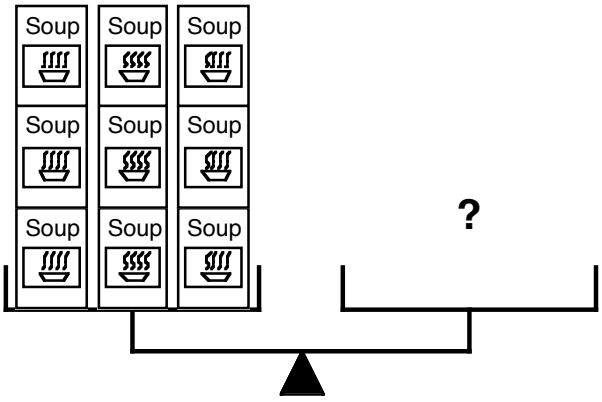
3. Write an equation that shows the relationship between x and y.

   _____

4. Use your equation to find the number of cans of beans that balance 9 cans of soup.

   _____

   Show your work.



6

Student D tries to combine like terms to form the equation in part three. Notice that the correct equation is under the diagram on the right. *What misconceptions does the student have about solving equations?* The student then uses the incorrect equation to try to solve for x.

Student D

The weight of one can of beans is x ounces.

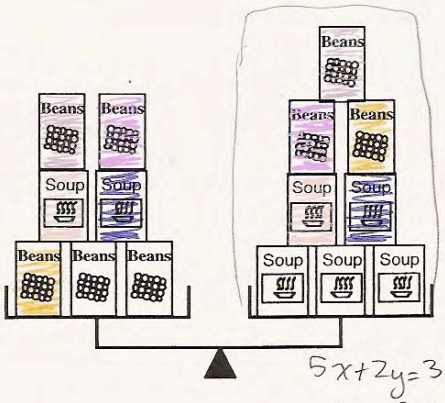
The weight of one can of soup is y ounces.

1. Write an expression for the weight of the cans on the left hand side of the weighing scales.

$5x+2y$ ✓ 1

2. Write an expression for the weight of the cans on the right hand side of the weighing scales.

✓ $3x+5y$ ✓ 1



3. Write an equation that shows the relationship between x and y.

X $7y=8x$ X 0 0

4. Use your equation to find the number of cans of beans that balance 9 cans of soup.

X

Show your work.

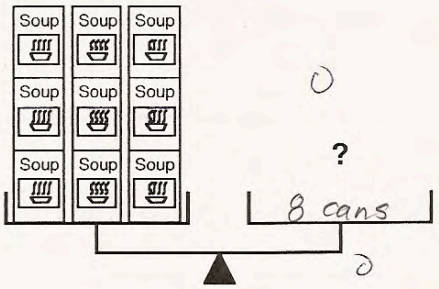
$7y=8x$

$\frac{7y}{7}=\frac{8x}{7}$

$\frac{63}{8}=\frac{8x}{8}$

$7,875=x$

8 cans of soup, because there are no fractions of cans of soup, so you round to 8.



2

2

Page 1

6

Soup and Beans

Student E has also tried to combine like terms and then generalize about the values. *Can you follow the reasoning that led to the equation x = y +1? What would be your next steps with this student?*

Student E

The weight of one can of beans is x ounces.

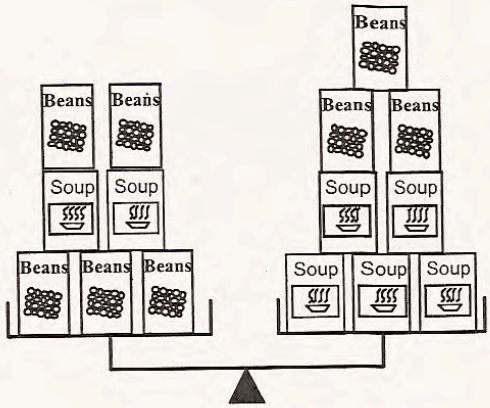
The weight of one can of soup is y ounces.

1. Write an expression for the weight of the cans on the left hand side of the weighing scales.

5x + 2y ✓ ✓

2. Write an expression for the weight of the cans on the right hand side of the weighing scales.

3x + 5y ✓ ✓



3. Write an equation that shows the relationship between x and y.

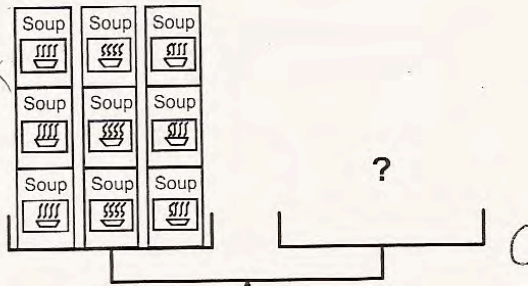
x=y+1 X X

4. Use your equation to find the number of cans of beans that balance 9 cans of soup.

10 X X

Show your work.

x = 10 X X

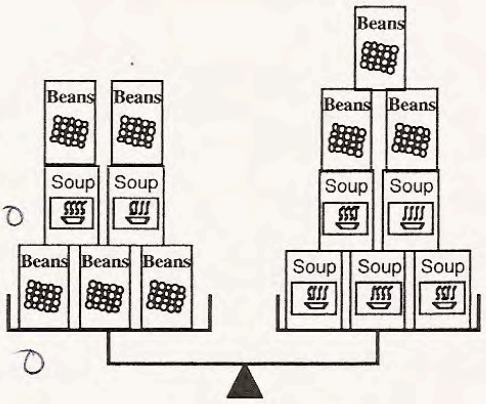


Student F writes the meaning of the variable instead of the expression in parts 1 and 2, but is able to build a correct equation in part 3. The student tries to use a table to solve for the cans of beans in part 5. Students often have trouble setting up tables for themselves, because too often the logic of designing the table is done by the textbook. Students pick up only pieces of the logic of making a table. *How could the table be modified to help the student find the correct number of cans?*

Student F

The weight of one can of beans is x ounces.

The weight of one can of soup is y ounces.



1. Write an expression for the weight of the cans on the left hand side of the weighing scales.

x= the weight of one can of beans.

2. Write an expression for the weight of the cans on the right hand side of the weighing scales.

y=the weight of one can of soup

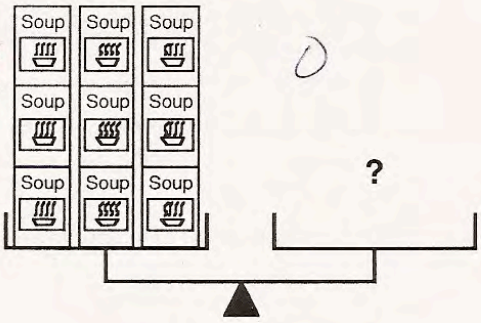
3. Write an equation that shows the relationship between x and y.

x·5+y·2 = x·3+y·5

4. Use your equation to find the number of cans of beans that balance 9 cans of soup.

Show your work.

x·5+y·2=x·3+y·5



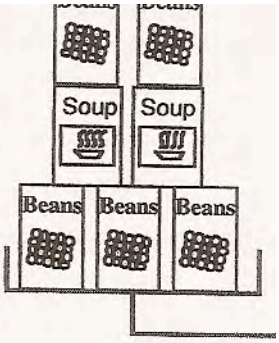
soup	9	y	9y
Beans	9	x	9x

Student G uses number sentences instead of writing expressions in part 1 and 2. This means that the equation will have a numerical value or be in a familiar form. *What is the conceptual understanding of equality that a student needs to make an equation from the diagram?*

Student G

1. Write an expression for the weight of the cans on the left hand side of the weighing scales.

X 5+2=7 O X



2. Write an expression for the weight of the cans on the right hand side of the weighing scales.

X 3+5=8 O X

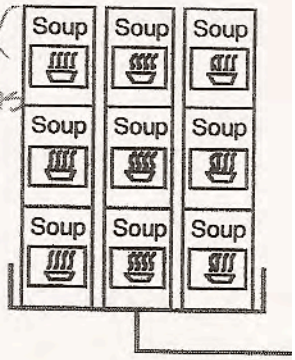
3. Write an equation that shows the relationship between x and y.

X 8x+7y=15 O

4. Use your equation to find the number of cans of beans that balance 9 cans of soup.

X 9 cans of beans

Show your work.



15=8x+7y

15=8(9)+7(9) X

15=72+63

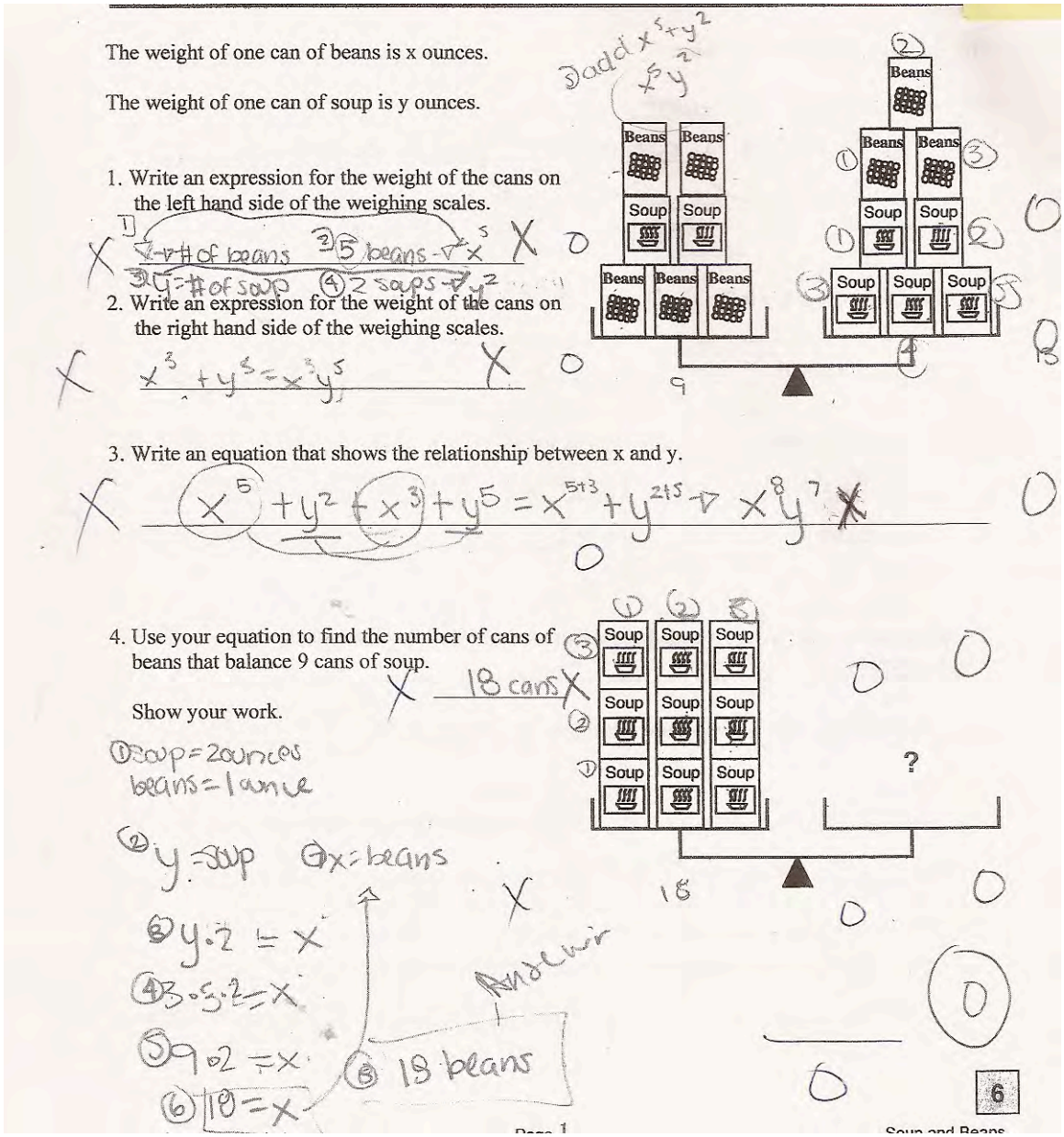
15=135 15|135 X

15 15 9cans

O

Student H does not understand algebraic notation and uses exponents in the expressions and equation. Notice that in part 4 the student assigns a value to the soup and beans rather than solving for x and y.

Student H



The weight of one can of beans is x ounces.

The weight of one can of soup is y ounces.

1. Write an expression for the weight of the cans on the left hand side of the weighing scales.

x = # of beans 5 beans = x^5

y = # of soup 2 soups = y^2

2. Write an expression for the weight of the cans on the right hand side of the weighing scales.

$x^3 + y^5 = x^3 y^5$

3. Write an equation that shows the relationship between x and y.

$x^5 + y^2 + x^3 + y^5 = x^{5+3} + y^{2+5}$ → $x^8 y^7$

4. Use your equation to find the number of cans of beans that balance 9 cans of soup.

Show your work.

18 cans

1 soup = 2 ounces
beans = 1 ounce

y = soup x = beans

$y \cdot 2 = x$

$3 \cdot 5 \cdot 2 = x$

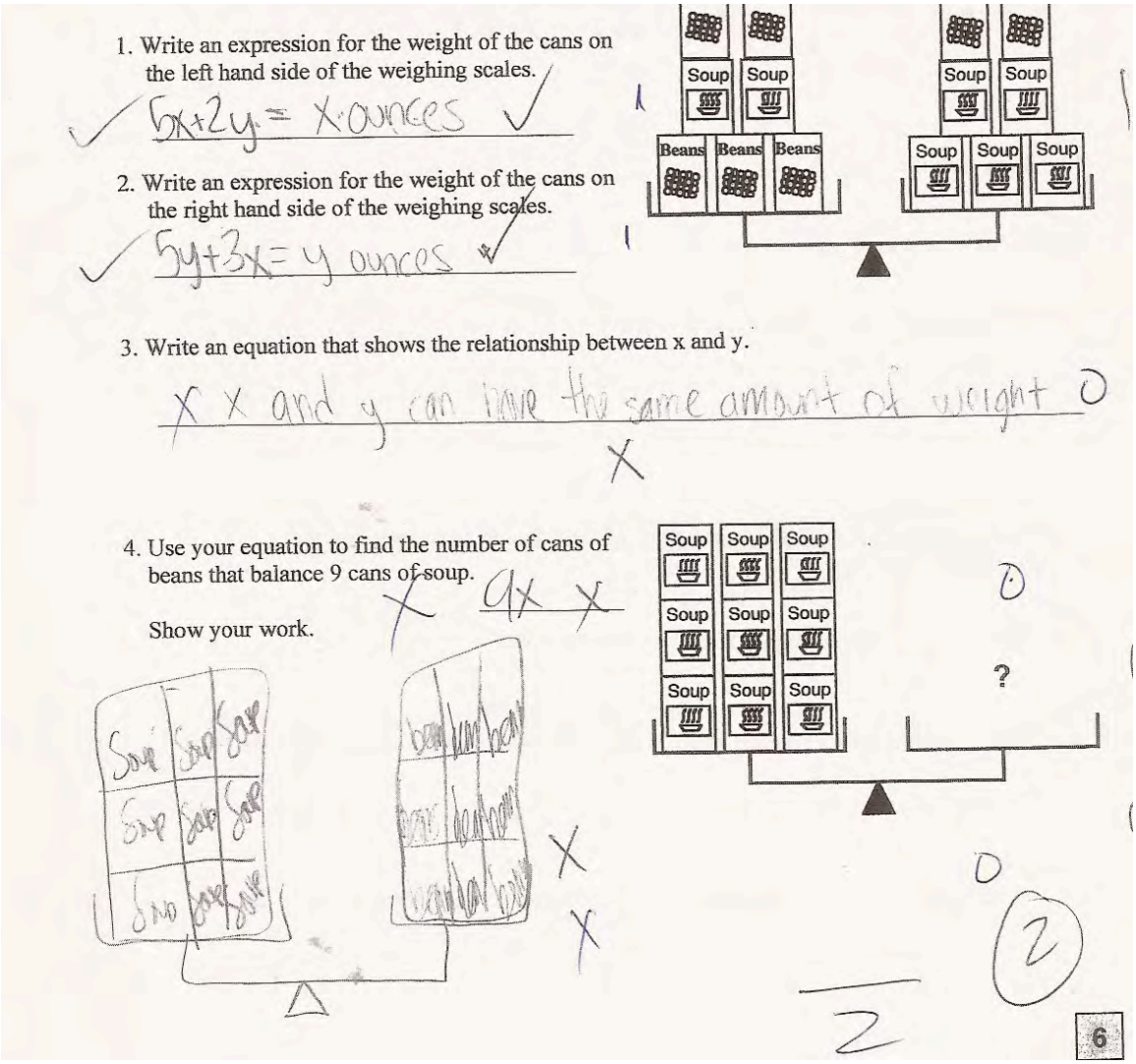
$9 \cdot 2 = x$

$18 = x$

18 beans

Student I makes the assumption that x and y are equal. *What question might you pose to get the student to re-examine his thinking? What evidence might the student notice that would contradict this idea?*

Student I



Algebra Task 1 Soup and Beans

Student Task	Make an equation and solve a problem.
Core Idea 3 Algebraic Properties and Representations	**Represent and analyze mathematical situations and structures using algebraic symbols.** • Understand the meaning of equivalent forms of expressions, equations, inequalities, or relations. • Write equivalent forms of equations, inequalities and systems of equations and solve them. • Use symbolic algebra to represent and explain mathematical relationships.

Mathematics of this task:

- Writing an expression from a diagram
- Understanding the relationship of equality in the context of a balanced scale
- Using equivalent ratios in context
- Distinguishing between a numerical coefficient and a value for the variable
- Understanding when there is sufficient information to solve for a variable
- Understanding meaning of variable or unknown

Based on teacher observations, this is what algebra students know and are able to do:

- Write expressions for diagrams
- Write equations for the diagram

Areas of difficulties for algebra students:

- Trying to assign values to x and y
- Trying to combine like terms from different sides of an equality
- Understanding the concept of equality illustrated in the diagram
- Using relational thinking to find equivalent ratios

MARS Test Task 1 Frequency Distribution and Bar Graph, Course 1

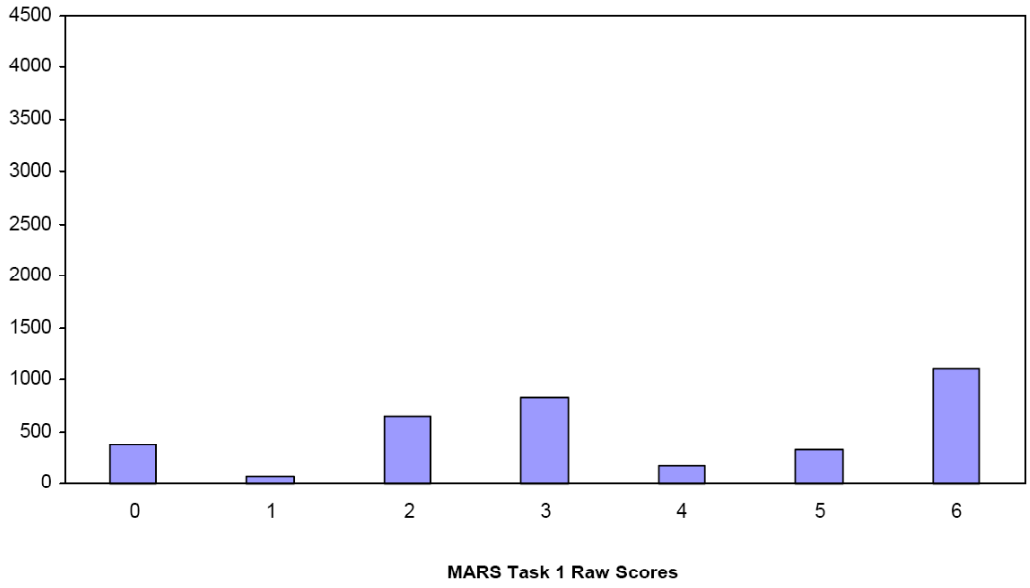
Task 1 - Soup and Beans

Mean: 3.61 StdDev: 2.02

Table 45: Frequency Distribution of MARS Test Task 1, Course 1

Task 1 Scores	Student Count	% at or below	% at or above
0	377	10.7%	100.0%
1	76	12.9%	89.3%
2	653	31.4%	87.1%
3	836	55.2%	68.6%
4	164	59.8%	44.8%
5	319	68.9%	40.2%
6	1096	100.0%	31.1%

Figure 54: Bar Graph of MARS Test Task 1 Raw Scores, Course 1



The maximum score available for this task is 6 points.
The minimum score needed for a level 3 response, meeting standards, is 3 points.

Most students, 87%, could write expressions for each side of the diagram. More than half the students, 69%, could also combine the expressions to make an equality statement about the diagram. 31% could meet all the demands of the task including using the new diagram and equivalent ratios to find the number of cans of beans to equal 9 cans of soup. Almost 11% of the students scored no points on this task. All of the students in the sample with this score attempted the task.

Soup and Beans

Points	Understandings	Misunderstandings
0	All of the students in the sample with this score attempted the task.	Students struggled with writing expressions from the diagram. Some students wrote number sentences, such as 5 + 2 = 7. Some students used exponents, such as $x^5 + y^2$. Some students tried to solve, giving answers like x=5 or y = 2.
2	Students could write expressions to represent the relationships in the diagram	Students struggled with writing the equation represented by the balance scale. 7% of the students wrote the expression 8x + 7y. Some students assumed that x = y or that x=y +1. Some students had equations with exponents. Others gave values for x and y.
3	Students could write expressions and equations for the situation depicted by the balance scale.	Students did not know how to combine the information in the diagram and the equation to find the number of cans of beans. 10% of the students thought the answer was 10. 10% thought the answer was 9. 8% thought the answer was 13.5. Many students got 2x = 3y and did no further work. 7% tried to substitute in 9 for y to get 2x = 27.
6	Students could write expressions and equations for the situation depicted by the balance scale. Students realized that 9 was the number of cans, not a value for y and could use equivalent ratios to find the number of cans of beans.	

Implications for Instruction

Students need more experiences using algebra to describe relationships. They should be able to understand and use equal value to write equations. Some students struggle with the basic concept of variable as a symbol describing a relationship between objects versus an answer.

Students need more opportunity to work with algebraic ideas in context. Students should be able to recognize equalities in situations, such as using a balance scale. Many students tried to combine the x's to equal the y's. They didn't think about the left side of the balance being equal to the right side of the balance.

Students need to understand the difference between a numerical coefficient and the value of the variable. Students saw the total of 8x and 7y and tried to define relationships with expressions like $x = y + 1$. Students were not thinking about multiplicative relationships. They also missed basic principals of symbolic manipulation, such as using inverse operations.

We want students to develop instincts about when there is enough information to solve for x or y and when there isn't enough information. This will help them when confronted with unusual problems like this one, where they needed instead to find equivalent ratios. Context is so important to developing a deeper conceptual understanding of algebraic principles because so many other ideas are brought into play that don't arise when just following procedures.

Ideas for Action Research

The Berkeley Lesson Study team at Willard Middle School developed a lesson on looking at numeric and symbolic strings. The Lesson, "Things that Mingle", was taught by Jacob Disston. The idea was to help students begin looking at the expressions and equations as mathematical objects with varying attributes.

This lesson is designed to push students to make connections between ideas about equations, inequalities, and expressions. The lesson gives students opportunities to use mathematical vocabulary for a purpose: to describe, discuss, and work with these number strings. In the process it is hoped that the students begin developing instincts about these symbolic, numeric strings, what might be used for. If this were on a test what might be the question? What is the purpose of looking at numeric strings as objects and gathering global information about the whole number string rather than thinking only about individual procedures or steps? Hopefully students will begin to see the number strings as mathematical objects with their own unique set of attributes. The lesson is designed to go across several standards in the algebra strand and to help students make connections across standards.

The lesson also explores various pedagogical strategies aimed at increasing student involvement, particularly from the African American and Latino students to help close the achievement gap. Students first need to walk around the room and interact with each to find people with number strings. Students are then asked to huddle at the board to discuss groupings. Cards are used for recording so that they can be moved easily if different students make different connections. Students are then placed in groups to have deeper conversations. They are also given opportunities to write and reflect as the lesson progresses.

Video, worksheets, discussion, student work, and a complete lesson plan are available on the Noyce website, Inside Mathematics: a professional learning community for educators. Sit down with colleagues and look at key parts of the lessons and lesson goals. Then discuss how the lesson changed the thinking of students. What changed about the attributes that they discussed? How were opportunities for conversation built into the lesson? How did features of the lesson increase the level of student participation? How might some of the ideas in this lesson help some of the struggles and misunderstandings that you saw in student work on this task.
www.Insidemathematics.org

2009 Rubrics

Soup and Beans	Rubric	
The core elements of performance required by this task are: • make an equation and solve a problem Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answer: **5x + 2y**	1	1
2. Gives correct answer: **3x + 5y**	1	1
3. Gives correct answers: **5x + 2y = 3x + 5y** or 2x = 3y	1	1
4. Gives correct answers: **6** Shows correct work such as: 2x = 3y 6x = 9y	2 1	3
Total Points		**6**

Soup and Beans

Work the task and look at the rubric. What are the key mathematical ideas being assessed? _____

Look at student work for part 1 and 2, writing expressions. How many of your students:

- Wrote correct expressions for the pictures?_____
- Wrote expressions with exponents?_____
- Wrote numbers sentences (no variables)?_____
- Gave values for x and y?_____
- Other?_____

What is confusing students about algebraic notation and the use of variables?

Now look at student work for part 3, writing an equation. How many of your students:

- Wrote a correct equation?_____
- Combined like terms (8x +7y)?_____
- Gave values for x and y?_____
- Used exponents?_____
- Other?_____

How do their misunderstandings about variable and notation effect their equations? What other misconceptions do you see as you examine their equations?

Now look at their work in part 4. How many of your students put:

6	13.5	9	10	18	7	Other

Now examine their strategies. What were some of the misconceptions that you noticed in their work?

What strategies did successful students use?

Looking at Student Work on Soup and Beans

Student A is able to write expressions and equations to represent the figures. The student is able to combine like terms in the equation using symbolic manipulation to find the equation 2x =3y. The student then realizes that he is not solving for x or y but looking for an equivalent ratio.

Student A

The weight of one can of beans is x ounces.

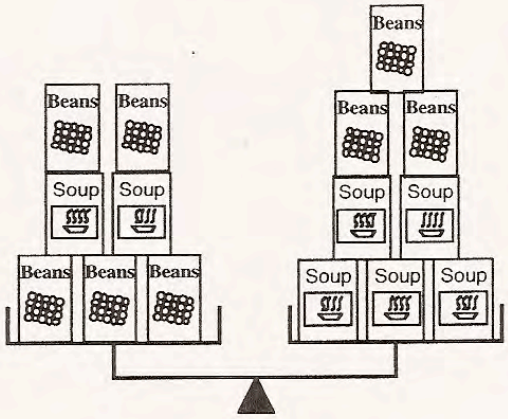
The weight of one can of soup is y ounces.

1. Write an expression for the weight of the cans on the left hand side of the weighing scales.

$5x+2y$

2. Write an expression for the weight of the cans on the right hand side of the weighing scales.

$3x+5y$



3. Write an equation that shows the relationship between x and y.

$5x+2y=3x+5y$

4. Use your equation to find the number of cans of beans that balance 9 cans of soup.

n=# of cans of beans

6 cans of beans

Show your work.

$nx=9y$

$5x+2y=3x+5y$

$-2y \quad -2y$

$5x=3x+3y$

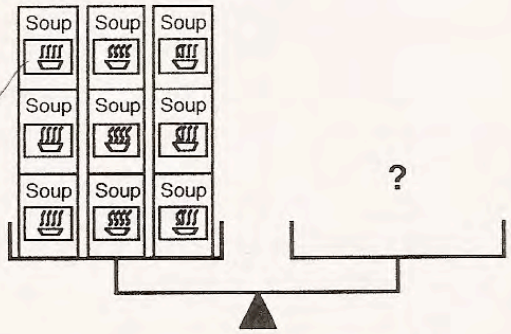
$-3x \quad -3x$

$2x=3y$

$2x=3y$

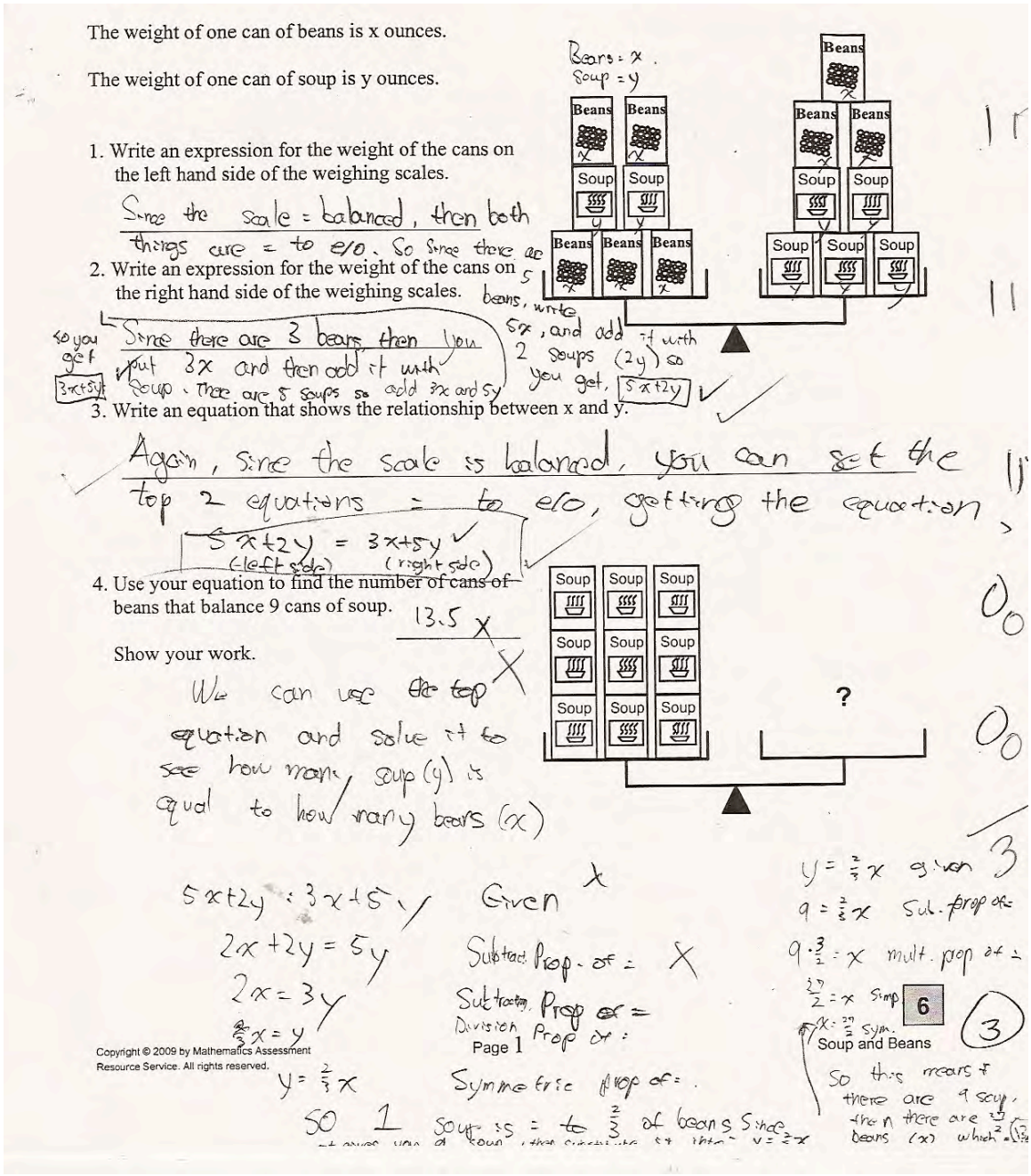
$\times 3$

$6x=9y$



Student B gives a complete description of how to write the expressions and equation. The student then finds the unit ratio, y =2/3 x. The student doesn't realize that the 9 is not the value of y, but the number of y units. So the 9 cannot be plugged into the equation to solve for x. This is a very subtle distinction. *What question can we pose in the classroom to help students explore this idea?*

Student B



Student C is able to write the expressions and equations. However the student seems to assign values to the soup and beans to reach a numeric value for x and y. The beans appear to be given a value 0.5. See the work in the top diagram. *Can you figure out what the student is doing?*

Student C

• make an equation and solve a problem

The weight of one can of beans is x ounces.

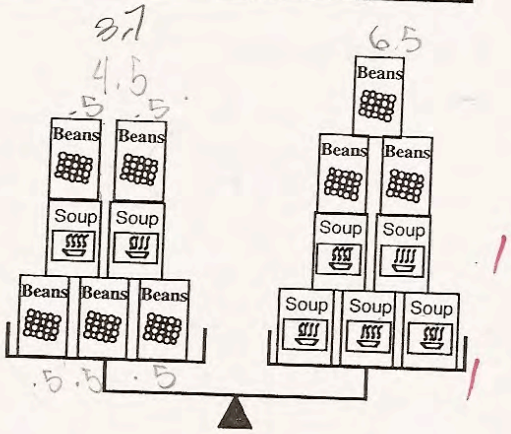
The weight of one can of soup is y ounces.

1. Write an expression for the weight of the cans on the left hand side of the weighing scales.

✓ $5x+2y=3x+5y$ ✓ f.w. 1

2. Write an expression for the weight of the cans on the right hand side of the weighing scales.

✓ $3x+5y$ ✓ 1



3. Write an equation that shows the relationship between x and y.

✓ $5x+2y=3x+5y$ ✓ 1 1

4. Use your equation to find the number of cans of beans that balance 9 cans of soup.

Show your work.